Math 54 Section Worksheet 7: Midterm Review

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Office Hours: Monday 3:30-5:30pm, Evans 1047
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1 Warm Up

Let A be an $n \times n$ matrix. Find someone next to you and together try to list as many properties that are equivalent to A being invertible. Make sure you are using the correct terminology when referring to matrices, subspaces, linear transformations, vectors, etc.

2 Problems

1. Let $\mathcal{B} = \{b_1, b_2\}$ and $\mathcal{C} = \{c_1, c_2\}$, where

$$b_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad b_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad c_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad c_2 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

   Suppose $x = 3b_1 + 2b_2$. Write x in the basis $\mathcal{C}$.

2. Let W be the set of all vectors of the form

$$\begin{pmatrix} -a+1 \\ a-6b \\ 2b+a \end{pmatrix}$$

   where a, b represent arbitrary real numbers. Either find a set S of vectors that spans W or give an example to show that W is not a subspace.

3. Calculate the determinant of $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ in two different ways: (a) by cofactor expansion, and (b) by row reducing.

4. Let $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$. Show that $\det A = 0$.

5. Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, and $\mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$.

   (a) Do $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^4$?

   (b) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation that projects onto the first 3 coordinates, i.e. it sends $(x_1, x_2, x_3, x_4) \mapsto (x_1, x_2, x_3)$. Does $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ span $\mathbb{R}^3$?

6. Which of the following is true?

   (a) A linearly independent set must contain the 0 vector.

   (b) A linearly independent set can never contain the 0 vector.

   (c) A linearly independent set can sometimes contain the 0 vector.

7. Which of the following is true?

   (a) A spanning set of a subspace must contain the 0 vector.

   (b) A spanning set of a subspace can never contain the 0 vector.

   (c) A spanning set of a subspace can sometimes contain the 0 vector.

8. Give an explicit example (don't just draw a picture) of a linear transformation that is:

   (a) both one-to-one and onto.

   (b) neither one-to-one nor onto.

   (c) one-to-one but not onto.

   (d) onto but not one-to-one.

9. Find bases for the column space and null space of $A = \begin{pmatrix} 2 & -4 & 6 & 0 & -6 \\ 0 & 0 & -2 & 12 & -10 \\ 1 & -2 & 4 & -5 & 1 \end{pmatrix}$.

10. Go back and do all the problems listed in all the old worksheets. Come to office hours. Ask me questions.