# Math 54 Section Worksheet 1 Solutions 

GSI: Jeremy Meza
Office Hours: TBD, Evans 1047
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## 1 Warm-Up

Try to recall the following concepts without looking at your notes.
augmented matrix elementary row operations row echelon form
reduced row echelon form pivot consistent linear system

## 2 Problems

1. Mark each statement True or False.
(a) Every elementary row operation is reversible. $\mathbf{T}$
(b) A $5 \times 6$ matrix has six rows. $\mathbf{F}$
(c) A solution of a linear system involving variables $x_{1}, \ldots, x_{n}$ is a list of numbers $\left(s_{1}, \ldots, s_{n}\right)$ that makes each equation in the system a true statement when the values $s_{1}, \ldots, s_{n}$ are substituted for $x_{1}, \ldots, x_{n}$, respectively. T
(d) Two fundamental questions about a linear system involve existence and uniqueness. $\mathbf{T}$
2. Mark each statement True or False.
(a) Two matrices are row equivalent if they have the same number of rows. $\mathbf{F}$
(b) Elementary row operations on an augmented matrix never change the solution set of the associated linear system. $\mathbf{T}$
(c) Two equivalent linear systems can have different solution sets. F
(d) A consistent system of linear equations has one or more solutions. $\mathbf{T}$
3. Mark each statement True or False.
(a) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations. F
(b) If one row in an echelon form of an augmented matrix is $\left[\begin{array}{lllll}0 & 0 & 0 & 5 & 0\end{array}\right]$, then the associated linear system is inconsistent. $\mathbf{F}$
(c) The row echelon form of a matrix is unique. $\mathbf{F}$
(d) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process. $\mathbf{F}$
(e) Whenever a system has free variables, the solution set contains many solutions. F
4. Row reduce the following matrix to reduced row echelon form:

$$
\left(\begin{array}{llll}
1 & 2 & 4 & 5 \\
2 & 4 & 5 & 4 \\
4 & 5 & 4 & 2
\end{array}\right)
$$

5. Do the three planes $2 x_{1}+4 x_{2}+4 x_{3}=4, x_{2}-2 x_{3}=-2$, and $2 x_{1}+3 x_{2}=0$ have at least one common point of intersection?
6. Suppose a $3 \times 5$ coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?
7. Suppose a system of linear equations has a $3 \times 5$ augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?
