Math 54 Quiz 4 Name: ______________________________
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1. True or False? You **must** justify your answer. *(2 points each).*

   (a) Let V be the set of matrices of the form $\begin{pmatrix} a & 1 \\ 0 & d \end{pmatrix}$ where $a, d \in \mathbb{R}$. Then, V is a vector space.

   (b) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. If $\operatorname{Ker} T = \mathbb{R}^n$, then T is one-to-one.

2. Let H be a subspace. Define what a basis of H is. *(1 point).*

3. For the following problem, we will let $\mathbb{P}_n$ denote the set of polynomials with real coefficients with degree **less than or equal to n**. Define a linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(p(t)) = \begin{pmatrix} p(0) \\ p'(0) \end{pmatrix}$.

   (a) Compute the image of $p(t) = 1 - 3t + t^2$. *(1 point).*

   (b) Find a matrix that represents T. *(2 points).*

   (c) Determine whether T is one-to-one, onto, both, or neither. *(2 points).*