

1. True or False? You **must** justify your answer. (2 points each).

(a) If A is a 3×4 matrix whose columns span \mathbb{R}^3 , then $Ax = 0$ has only the trivial solution.

(b) If A is a 4×3 matrix, then A has linearly independent columns.

(c) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in \mathbb{R}^3 and $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \mathbb{R}^3$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent.

(d) If A is a 2×3 matrix with 2 pivot positions, then there exists a solution to $Ax = b$ for every $b \in \mathbb{R}^2$.

2. Let $v_1 = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix}$, $v_3 = \begin{pmatrix} -5 \\ -8 \\ 9 \end{pmatrix}$, and $b = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Determine if b is in the span of

v_1, v_2, v_3 . If so, describe in parametric form all the vectors $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ such that $c_1v_1 + c_2v_2 + c_3v_3 = b$. (2 points.)