Math 54 Midterm 1 Review

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1 Bases and Spaces

1. Find bases for the column space and null space of $A = \begin{pmatrix} 2 & -4 & 6 & 0 & -6 \\ 0 & 0 & -2 & 12 & -10 \\ 1 & -2 & 4 & -5 & 1 \end{pmatrix}$.

2. Find bases for the row space and the left null space of $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & -9 & 7 \\ -1 & -3 & 2 \\ 2 & 6 & -5 \end{pmatrix}$.

3. You have a homogenous system of 18 linear equations and 20 variables. You want to conclude that every associated nonhomogeneous system of equations has some solution. What must be true about the solution set of the original homogenous system for you to be able to conclude this?

4. Let $\mathbf{b} = \begin{pmatrix} 1 \\ -4 \\ -9 \end{pmatrix}, \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$, and $\mathbf{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$. Determine if $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

5. Let $\mathcal{B} = \{b_1, b_2\}$ and $\mathcal{C} = \{c_1, c_2\}$, where

$$b_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad b_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad c_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad c_2 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

   Suppose $x = 3b_1 + 2b_2$. Write x in the basis $\mathcal{C}$. What is the change of coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$ and vice-versa, from $\mathcal{C}$ to $\mathcal{B}$?

6. Which of the following is true?
   (a) A basis B must contain the 0 vector.
   (b) A basis B can never contain the 0 vector.
   (c) A basis B can sometimes contain the 0 vector, depending on the subspace H.

7. Give an explicit example (don't just draw a picture) of a linear transformation that is:
   (a) both one-to-one and onto.
   (b) neither one-to-one nor onto.
   (c) one-to-one but not onto.
   (d) onto but not one-to-one.

2 Determinants and Inverses

1. Calculate the determinant of $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ in two different ways: (a) by cofactor expansion, and (b) by row reducing.

2. Calculate the inverse of $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & -2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$.

3. Let $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$. Show that $\det A = 0$.

4. Use Cramer's rule to solve the following system of linear equations:

$$\begin{aligned} x + y + z &= 11 \\ 2x - 6y - z &= 0 \\ 3x + 4y + 2z &= 0 \end{aligned}$$

5. Let A be an $n \times n$ matrix. Without looking at the book or your notes, try to list as many properties of A that are equivalent to A being invertible.

3 Type Inference & Miscellany

1. Below is a list of nonsensical statements that you (yes you) has used. Go through each one and understand why it doesn't quite make sense.
   (a) A basis for the matrix A.
   (b) The linear transformation is linearly independent.
   (c) The column space of the equation $Ax = 0$.
   (d) The kernel of the matrix.
   (e) The subspace is a basis.
   (f) There is a pivot in every row of the linear transformation T.
   (g) The matrix A is onto.
   (h) The rank of the vectors is 5.
   (i) $Ax = b$ has a solution for each x

2. Go back and do all the green problems listed in all the old worksheets.

3. Set a timer for 1 hour. Do the practice midterm.