Math 54 Section Worksheet 7

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1 Green Problems

1. (4.6 # 10). If the null space of a 7×6 matrix A is 5-dimensional, what is the dimension of the column space of A?

2. (4.6 # 11). If the null space of an 8×5 matrix A is 2-dimensional, what is the dimension of the row space of A?

3. (4.6 # 12). If the null space of a 5×6 A is 4-dimensional, what is the dimension of the row space of A?

4. (4.6 # 13). If A is a 7×5 matrix, what is the largest possible rank of A? If A is a 5×7 matrix, what is the largest possible rank of A? Explain.

5. (4.6 # 14). If A is a 4×3 matrix, what is the largest possible dimension of the row space of A? If A is a 3×4 matrix, what is the largest possible dimension of the row space of A? Explain.

6. (4.6 # 15). If A is 6×8 matrix, what is the smallest possible dimension of Nul A?

7. (3.2 #27). True or False.

   (a) A row replacement operation does not affect the determinant of a matrix.

   (b) The determinant of A is the product of the pivots in any echelon form U of A, multiplies by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.

   (c) If the columns of A are linearly dependent, then $\det A = 0$.

   (d) $\det(A + B) = \det A + \det B$.

8. (3.2 #28). True or False.

   (a) If three row interchanges are made in succession, then the new determinant equals the old determinant.

   (b) The determinant of A is the product of the diagonal entries in A.

   (c) If $\det A$ is zero, then two rows or two columns are the same, or a row or column is zero.

   (d) $\det A^{-1} = -\det A$.

2 Extra Problems

9. Let $\mathcal{B} = \{b_1, b_2\}$ and $\mathcal{C} = \{c_1, c_2\}$. Find the change of coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$.

$$b_1 = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad b_2 = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \quad c_1 = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad c_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

10. Let A be a $p \times q$ matrix. Which of the subspaces $\operatorname{Row} A, \operatorname{Col} A, \operatorname{Nul} A, \operatorname{Row} A^T, \operatorname{Col} A^T$ and $\operatorname{Nul} A^T$ are in $\mathbb{R}^p$ and which are in $\mathbb{R}^q$? How many distinct subspaces are in this list?

3 Challenge

11. Show that the space $C(\mathbb{R})$ of all continuous functions defined on the real line is an infinite dimensional space.