Math 54 Section Worksheet 6

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1 Green Problems

1. (4.5 # 19). True or False.
   (a) The number of pivot columns of a matrix equals the dimension of its column space.
   (b) A plane in $\mathbb{R}^3$ is a two-dimensional subspace of $\mathbb{R}^3$.
   (c) The dimension of the vector space $\mathbb{P}_4$ is 4.
   (d) If $\dim V = n$ and S is a linearly independent set in V, then S is a basis for V.
   (e) If a set $\{v_1, \ldots, v_p\}$ spans a finite-dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent.
2. (4.5 # 20). True or False.
   (a) $\mathbb{R}^2$ is a two-dimensional subspace of $\mathbb{R}^3$.
   (b) The number of variables in the equation $Ax = 0$ equals the dimension of Nul A.
   (c) A vector space is infinite-dimensional if it is spanned by an infinite set.
   (d) If $\dim V = n$ and if S spans V, then S is a basis of V.
   (e) The only three-dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself.
3. (4.6 # 10). If the null space of a 7×6 matrix A is 5-dimensional, what is the dimension of the column space of A?
4. (4.6 # 11). If the null space of an 8×5 matrix A is 2-dimensional, what is the dimension of the row space of A?
5. (4.6 # 12). If the null space of a 5×6 A is 4-dimensional, what is the dimension of the row space of A?
6. (4.6 # 13). If A is a 7×5 matrix, what is the largest possible rank of A? If A is a 5×7 matrix, what is the largest possible rank of A? Explain.
7. (4.6 # 14). If A is a 4×3 matrix, what is the largest possible dimension of the row space of A? If A is a 3×4 matrix, what is the largest possible dimension of the row space of A? Explain.
8. (4.6 # 15). If A is 6×8 matrix, what is the smallest possible dimension of Nul A?

2 Extra Problems

9. Let A be a $p \times q$ matrix. Which of the subspaces $\operatorname{Row} A, \operatorname{Col} A, \operatorname{Nul} A, \operatorname{Row} A^T, \operatorname{Col} A^T$ and $\operatorname{Nul} A^T$ are in $\mathbb{R}^p$ and which are in $\mathbb{R}^q$? How many distinct subspaces are in this list?

3 Challenge

10. Show that the space $C(\mathbb{R})$ of all continuous functions defined on the real line is an infinite dimensional space.