Math 54 Section Worksheet 5

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1 Green Problems

1. (2.1 # 23). Suppose $CA = I_n$ (the $n \times n$ identity matrix). Show that the equation $Ax = 0$ has only the trivial solution. Explain why A cannot have more columns than rows.

2. (2.1 # 24). Suppose $AD = I_m$ (the $m \times m$ identity matrix). Show that for any b in $\mathbb{R}^m$, the equation $Ax = b$ has a solution. Explain why A cannot have more rows than columns.

3. (2.3 # 11). All matrices below are $n \times n$. Mark each implication as True or False.

   (a) If the equation $Ax = 0$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix.

   (b) If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent.

   (c) If A is an $n \times n$ matrix, then the equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$.

   (d) If the equation $Ax = 0$ has a nontrivial solution, then A has fewer than n pivot positions.

   (e) If A^T is not invertible, then A is not invertible.

4. (2.2 # 9). Mark each statement as True or False.

   (a) In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true.

   (b) If A and B are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of AB.

   (c) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ab - cd \neq 0$, then A is invertible.

   (d) If A is an invertible $n \times n$ matrix, then the equation $Ax = b$ is consistent for each b in $\mathbb{R}^n$.

   (e) Each elementary matrix is invertible.

5. (2.3 # 15). Can a square matrix with two identical columns be invertible? Why or why not?

2 Extra Problems

6. If the equation $Hx = c$ is inconsistent for some c in $\mathbb{R}^n$, what can you say about the equation $Hx = 0$?

7. Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of A are linearly independent.

8. Suppose $AB = \begin{pmatrix} 5 & 4 \\ -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$. Find A.

3 Challenge

9. (a) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Show that $A^3 = 0$. Compute $(I - A)(I + A + A^2)$.

   (b) Suppose $A^n = 0$ for some $n > 1$. Find an inverse for $I - A$.

10. Prove that there exists 2×2 matrices A, B such that $C = AB - BA$ if and only if $c_{11} + c_{22} = 0$.

11. If A is a 2×1 matrix and B is a 1×2 matrix, prove that $C = AB$ is not invertible.