Math 54 Section Worksheet 4

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September 6, 2018

1 Green Problems

1. (1.9 # 23). Mark each statement True or False.
   (a) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity matrix.
   (b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates vectors about the origin through an angle φ, then T is a linear transformation.
   (c) When two linear transformations are performed one after another, the combined effect may not always be a linear transformation.
   (d) A mapping $T : \mathbb{R}^n \to R^m$ is onto R^m if every vector x in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.
   (e) If A is a 3×2 matrix, then the transformation $x \mapsto Ax$ cannot be one-to-one.

2. (1.9 # 24). Mark each statement True or False.
   (a) Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.
   (b) The columns of the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the columns of the $n \times n$ identity matrix.
   (c) The standard matrix of a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects points through the horizontal axis, the vertical axis, or the origin has the form $\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$, where a and d are ± 1.
   (d) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$.
   (e) If A is a 3×2 matrix, then the transformation $x \mapsto Ax$ cannot map $\mathbb{R}^2$ onto $\mathbb{R}^3$.

2 Extra Problems

3. Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, y_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, and $y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps e_1 to y_1 and maps e_2 to y_2. Find the images of $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

4. Prove that every linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation. That is, construct a matrix A such that $T(x) = Ax$ for all $x \in R^n$. (Hint: look what T and A do to basis elements.)

5. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one. Describe the possible echelon forms of its corresponding matrix.

6. Suppose $T : \mathbb{R}^4 \to \mathbb{R}^3$ is onto. Describe the possible echelon forms of its corresponding matrix.