Math 54 Section Worksheet 2

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1 Green Problems

1. (1.2 # 21). Mark each statement True or False.
   (a) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.
   (b) The row reduction algorithm applies only to augmented matrices for a linear system.
   (c) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.
   (d) Finding a parametric description of the solutions set of a linear system is the same as solving the system.
   (e) If one row in an echelon form of an augmented matrix is $\begin{bmatrix}0 & 0 & 0 & 5 & 0\end{bmatrix}$, then the associated linear system is inconsistent.

2. (1.2 # 22). Mark each statement True or False.
   (a) The echelon form of a matrix is unique.
   (b) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.
   (c) Reducing a matrix to echelon form is called the forward phase of the row reduction process.
   (d) Whenever a system has free variables, the solution set contains many solutions.
   (e) A general solution of a system is an explicit description of all solutions of the system.

3. (1.2 # 23). Suppose a 3×5 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

4. (1.2 # 24) Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot oclumn. Is the system consistent? Why or why not?

5. (1.3 # 24). Mark each statement True or False.
   (a) Any list of five real numbers is a vector in $\mathbb{R}^5$.
   (b) The vector $\mathbf{u}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$.
   (c) The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v_1} + \cdots + c_p\mathbf{v_p}$ cannot all be zero.

(d) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

(e) Asking whether the linear system corresponding to an augmented matrix $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ has a solution amounts to asking whether $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

6. (1.5 # 29-32). (a) Does the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution and (b) does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

   (a) A is a 3×3 matrix with three pivot positions.

   (b) A is a 3×3 matrix with two pivot positions.

   (c) A is a 3×2 matrix with two pivot positions.

   (d) A is a 2×4 matrix with two pivot positions.

2 Extra Problems

7. Find the general solutions of the system whose augmented matrix is

$$\begin{pmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{pmatrix}$$

8. Describe the solutions of the following system in parametric vector form. Give a geometric description of the solution set.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

9. Think of questions to ask!

3 Challenge

10. Suppose $A\mathbf{x} = \mathbf{b}$ has a solution. Explain why the solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

11. Construct a 2×2 matrix A such that the solution set of the equation $A\mathbf{x} = \mathbf{0}$ is the line in $\mathbb{R}^2$ through $(4, 1)$ and the origin. Then, find a vector $\mathbf{b}$ in $\mathbb{R}^2$ such that the solution set of $A\mathbf{x} = \mathbf{b}$ is not a line in $\mathbb{R}^2$ parallel to the solution set of $A\mathbf{x} = \mathbf{0}$.