Math 54 Section Worksheet 15

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1 Recall

1. Write down a formula for e^{At}, where A is an $n \times n$ matrix.

2. Find eigenvalues and eigenvectors for the matrix $A = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$.

3. (a) Find two solutions $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ to the system

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix} \mathbf{x}(t)$$

   (b) Check that your solutions are linearly independent.

2 Solving Homogeneous Linear Systems

Consider the system

$$\mathbf{x}'(t) = \mathbf{A}(t)\mathbf{x}(t)$$

where $\mathbf{A}(t)$ is an $n \times n$ matrix.
Question: How do we find all solutions to this linear system?

A *fundamental solution set* is ____.

A *fundamental matrix* is ____.

All solutions to the above linear system are of the form

$$\mathbf{x}(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

If $\mathbf{A}(t) = A$ is a constant matrix, then e^{At} is __________________________________.

Answer: If $\mathbf{A}(t) = A$ is constant, then all solutions to the linear system will be linear combinations of the columns of e^{At}.

So, how do we compute e^{At}?

<u>Case 1:</u> A is diagonal, say $A = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_n \end{pmatrix}$.

Then, $e^{At} =$

Case 2: A is diagonalizable, say $A = PDP^{-1}$.

Then, $e^{At} =$

You Try: Compute e^{At} where $A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$. (if it helps, $\lambda = 4$ is an eigenvalue with eigenvector $(1, 1, 1)$.)

3 Solving Nonhomogeneous Linear Systems

Consider the system

$$\mathbf{x}'(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{f}(t)$$

Again, to simplify, we will assume $\mathbf{A}(t)$ is a constant matrix.
Question: How do we find all solutions to this linear system?
All solutions to the above linear system are of the form

$\mathbf{x}(t) =$ ____

Answer: We know how to find homogeneous solutions, so to find a particular solution, we guess!

Let's find a general solution to $\mathbf{x}'(t) = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} -t - 1 \\ -4t - 2 \end{pmatrix}$.

You Try: Find a general solution to $\mathbf{x}'(t) = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} -4\cos t \\ -\sin t \end{pmatrix}$.