Math 54 Midterm 2 Review

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1 Test Taking Tips

Below are some strategies to keep in mind tomorrow (and for future math tests):

- Read the statement prompt.
  - What is the question asking for? Is it asking for a vector? A scalar? Make sure whatever answer you give is the correct *type* of answer.
  - Did you answer *all* that the question was asking? Don't forget anything!
- Copy numbers over correctly. Some of the most common errors are just doing the problem with the wrong set of numbers.
- Take your time with arithmetic. Feel free to write mental calculations down in the margins.
- Write your solutions as if someone is reading them (because someone is reading them!) For example, do any scratch work on one part of the paper and then try to make it clear what thought processes are going on through your head.
  - The strategy of putting everything you can think of on the paper in chaotic fashion hoping for some partial credit does *not* work.
- Check your answer. At the basic level, this means going through your answer and making sure you didn't make any mistakes. More than this, make sure your answer satisfies whatever properties it *should* satisfy.
  - For example, if you take your orthogonal projection and subtract it from your given vector, is it orthogonal to what it should be? Check any orthonormal basis you find is in fact orthonormal. If you find an eigenvalue and eigenvector, does it satisfy $Av = \lambda v$?

2 Eigenvalues, Eigenvectors, Diagonalizability

1. Let $A = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$. Is A diagonalizable?
2. Let $A = \begin{pmatrix} 1 & 1 & -5 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$. Find the eigenvalues of A. For each eigenvalue, find a basis for the corresponding eigenspace.

3 Orthogonality, Gram-Schmidt, Least Squares

1. Let $\mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$ and let $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 1 & -1 \\ 0 & -1 & -3 \end{pmatrix}$.

   (a) Find an orthonormal basis for the column space of A.

   (b) Find a least squares solution to $A\mathbf{x} = \mathbf{b}$.

   (c) Find the shortest distance from $\mathbf{b}$ to the column space of A.

4 Spectral Theorem

1. (7.1 #25) True or False.

   (a) An $n \times n$ matrix that is orthogonally diagonalizable must be symmetric.

   (b) If $A^T = A$ and if vectors $\mathbf{u}$ and $\mathbf{v}$ satisfy $A\mathbf{u} = 3\mathbf{u}$ and $A\mathbf{v} = 4\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = 0$.

   (c) An $n \times n$ symmetric matrix has n distinct real eigenvalues.

   (d) For a nonzero $\mathbf{v}$ in $\mathbb{R}^n$, the matrix $\mathbf{v}\mathbf{v}^T$ is called a projection matrix.

2. (7.1 #26) True or False.

   (a) There are symmetric matrices that are not orthogonally diagonalizable.

   (b) If $B = PDP^T$, where $P^T = P^{-1}$ and D is a diagonal matrix, then B is a symmetric matrix.

   (c) An orthogonal matrix is orthogonally diagonalizable.

   (d) The dimension of an eigenspace of a symmetric matrix is sometimes less than the multiplicity of the corresponding eigenvalue.