Math 54 Section Worksheet 13

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1 Green Problems

4. (6.5 #17) True or False. (A is a $m \times n$ matrix and $\mathbf{b}$ is in $\mathbb{R}^m$).
   (a) The general least-squares problem is to find an $\mathbf{x}$ that makes $A\mathbf{x}$ as close as possible to $\mathbf{b}$.
   (b) A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a vector $\hat{\mathbf{x}}$ that satisfies $A\hat{\mathbf{x}} = \hat{\mathbf{b}}$, where $\hat{\mathbf{b}}$ is the orthogonal projection of $\mathbf{b}$ onto $\operatorname{Col} A$.
   (c) A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a vector $\hat{\mathbf{x}}$ such that $\|\mathbf{b} - A\mathbf{x}\| \leq \|\mathbf{b} - A\hat{\mathbf{x}}\|$ for all $\mathbf{x}$ in $\mathbb{R}^n$.
   (d) Any solution of $A^TA\mathbf{x} = A^T\mathbf{b}$ is a least-squares solution of $A\mathbf{x} = \mathbf{b}$.
   (e) If the columns of A are linearly independent, then the equation $A\mathbf{x} = \mathbf{b}$ has exactly one least-squares solution.
5. (6.5 #18) True or False. (A is a $m \times n$ matrix and $\mathbf{b}$ is in $\mathbb{R}^m$).
   (a) If $\mathbf{b}$ is in the column space of A, then every solution of $A\mathbf{x} = \mathbf{b}$ is a least squares solution.
   (b) The least-squares solution of $A\mathbf{x} = \mathbf{b}$ is the point in the column space of A closest to $\mathbf{b}$.
   (c) A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a list of weights that, when applied to the columns of A, produces the orthogonal projection of $\mathbf{b}$ onto $\operatorname{Col} A$.
   (d) If $\hat{\mathbf{x}}$ is a least-squares solution of $A\mathbf{x} = \mathbf{b}$, then $\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{b}$.
   (e) The normal equations always provide a reliable method for computing least-squares solutions.
   (f) If A has a QR factorization, say $A = QR$, then the best way to find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ is to compute $\mathbf{x} = R^{-1}Q^T\mathbf{b}$.
6. Find someone with a book and go through Supplementary 1.