Math 55 Section 101, M 2/11/16: Sets & Functions

Q: What event of great political significance is today?
A: Iowa Caucus!

Consider following electoral situations:
A - Heavily evangelical (Christian/Religious) district.
B - Rural district, all Farms, Farmers, & Farm workers.

A: Some hypothetical "facts":
- Ted Cruz is campaigning entirely to religious voters
- Evangelical voters thus overwhelmingly support Cruz
- Bush, Rubio, Campaign to moderates/non-religious conservatives.
- Evangelicals are 30% of residents in district A
- But! Evangelicals vote 90% of time, secular conservatives don’t generally vote.
- Who wins? Obviously Cruz.

Ven Diagram
Crut Trump Bush etc.

Iowa Populations
May be not so bad for non-Cruz candidates...

- actual votes

Uh-oh.

Venn diagrams are helpful...
More "facts"

- All adults in district B work in agriculture.
- Farm workers support Sanders (yay healthcare).
- Farm owners support Hillary (more pro-business).
- Who wins the district?
- Probably Sanders. Why?

- Every worker has a boss (farm owner).
- Every owner probably employs >1 worker.

\[ \implies \text{More workers than owners.} \]

**Translation:**

\[ F := \text{set of farm workers}, \quad \Omega := \text{set of owners} \]

\[ W : F \to \Omega. \text{ Function } w \text{ sends farm worker to owner of his workplace. Map is surjective, not injective} \]

\[ \implies \text{size of } F \geq \text{size of } \Omega \]

Knowing about injection/surjection/bijection between sets gives info about relative size. (For finite sets.)

Also sort-of for oo sets.

More mathy example:

A partition of an integer (negative) is an expression of \( n \) as a sum of other non-negative integers. Order doesn't matter. (Positive)

Examples: partition into 4 #5

(a) \( 10 = 5 + 3 + 1 + 1 \) (same as \( 1 + 5 + 3 + 1 \) or \( 3 + 1 + 1 + 5 \))

(b) \( 25 = 10 + 5 + 5 + 2 + 1 + 1 + 1 \) (repeats are allowed)
**Question:**

How many ways to partition \( n \) into \( \leq k \) #s each \( \leq j \)?

**Example:**

(a) \( 10 = 3 + 3 + 2 + 2 \) is partition into \( \leq 4 \) #s, each \( \leq 10 \) (i.e., \( 3, 2 \leq 10 \))

(b) \( 11 = 2 + 2 + 2 + 1 + 1 + 1 \) is partition into \( \leq 10 \) #s, each \( \leq 2 \).

Actually hard formula, not even sure if there is a general one...

**BUT** we can show:

**Thm:** \# of ways to partition \( n \) into \( sk \) #s each \( \leq j \)  

\[ \text{equals} \]  

\# of ways to partition \( n \) into \( sj \) #s each \( \leq sk \)

\( \Rightarrow \) can switch \( k \) with \( j \) and get same number.

Do this with clear bijection!

---

**Proof of Thm**

\( S := \text{set of ways to partition } n \text{ into } \leq sk \#s \text{ each } \leq j \)

\( T := \text{set of ways to partition } n \text{ into } \leq sj \#s \text{ each } \leq sk \)

Can express partition in \( S \) as "table of boxes":

\[ 10 = 5 + 3 + 1 + 1 \Rightarrow \hspace{1cm} \begin{array}{c|c|c|c|c} \hline & & & & \hline \end{array} \begin{array}{c} \hline 5 \hline \end{array} \begin{array}{c} \hline 3 \hline \end{array} \begin{array}{c} \hline 1 \hline \end{array} \begin{array}{c} \hline 1 \hline \end{array} \hspace{1cm} \Rightarrow \hspace{1cm} \text{can do with any } n \text{ boxes} \]

total of 10 boxes
Partitions of \( n \) into \( \leq j \) #s each \( \leq j \) have tables like so:

- From \( S \) to \( T \):
  - \( \Rightarrow \) \( \leq j \) rows \( \leq k \) boxes
  - \( \Rightarrow \) \( \leq j \) #s in \( S \)
  - \( \Rightarrow \) \( \leq j \) #s in \( S \)

There is a "Flip" function \( F: S \rightarrow T \) giving bijection.

\[ \Rightarrow \text{size of } S \text{ equals size of } T! \]

Lesson: Bijection tells you #s/sites of sets are equal even when you don't know actual sizes! Cool