Section 3/31 Dis Template

Tuesday, March 31, 2020 12:49 PM

Announcements:

- Quiz will be **tomorrow at 2 pm PST**
- Administered on BCourses. **30 minute time limit**, 10 minute grace period at the end to turn it in.
- No proctor, time limit applies to submission.
- Sorry about my earlier announcement!

Topics Today: Sequences (12.1), Annuities (12.2) and Series (12.4).

<u>Sequences</u>

A **sequence** a_n is literally a sequence of numbers a_1, a_2, a_3 ... Here a_n is called the **n-th term**. The **Nth partial sum** $\boldsymbol{S_N}$ of a series a_n is the sum

$$\boldsymbol{S_N} = \sum_{n=1}^{N} a_n$$

The most important example for Chapter 12 are **geometric sequences**. These are sequences of the form:

$$a_n = ar^{n-1} \text{ for numbers } a \text{ and } r$$

Example: the sequence 3, 6, 12, 24, 48 … is just the geometric sequence ar^{n-1} for $a = 3$ and $r = 2$. The 1st partial sum is 3, the 2nd partial sum is 3 + 6 = 9, the 3r is 3 + 6 + 12 = 21 and so on.

Important Formula: The **Nth partial sum** of a geometric series is given by the formula

$$\sum_{n=1}^{N} a_n = \sum_{n=1}^{N} ar^{n-1} = a\frac{r^N - 1}{r - 1}$$

Exercises:

1) List the first 3 terms of the geometric sequence with $a = 5$ and $r = 3$.

2) Find the 4th partial sum of that geometric series.

3) Compute the partial sum $\sum_{n=1}^{30} 30(\frac{1}{3})^{n-1}$. Use a calculator.

Annuities

Some Vocab: An **annuity** is a regular payment at equal periods of time. The **payment period** is… that equal period of time. The **term** is how long between the first and last payment.

The **amount of the annuity** is the final amount of money that you end up with after the term is up. The **present value of the annuity** is the lump sum you should invest at the given interest rate at the beginning of the term to get an amount of money at he end equal to the final amount.

Example Question: An annuity pays $1500 yearly into an account paying 8% interest per year, compounded annually. What is the total annuity (i.e. the final amount)?

We want formulas for the amount and present value.

There is a formula for the amount. What is it?

Important Formulas: Consider an annuity consisting of annual payments of $\$R$ per period, for n consecutive interest periods with interest i (decimal interest, not percent).

Then the amount of annuity S is given by

$$S = R\frac{(1+i)^n - 1}{i}$$

The present value P of an annuity is given by

$$P = (1+i)^n S$$

Exercise: Check that the second formula is the same as

$$P = R\,\frac{1-(1+i)^{-n}}{i}$$

Exercises:

1) Use formula to find the amount and present value of an annuity paying $9000 yearly for 18 payment periods, at an interest of 6%.

Series

An **infinite series** is an infinite sum of the form

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots.$$

The very precise thing to say is that this infinite sum is the limit of the partial sums

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{n=1}^{N} a_n$$

as N goes to infinity. If this limit exists, then we say that the infinite series **converges.** If the limit doesn't exist (e.g. the partial sums go to infinity).

Example: If $a_n = ar^{n-1}$ is a geometric series, then the partial sums are given by

$$S_N = a\frac{r^N - 1}{r - 1}$$

if $r \geq 1$, then as N goes to infinity, the formula above gets very large and diverges. Otherwise the r^N factor goes to 0 as N goes to infinity an we get

Important Formula: If $a_n = ar^{n-1}$ is a geometric series, then

$$\sum_{n=1}^{\infty} a_n = \frac{a}{1-r} \quad \text{if} \quad |r| < 0$$

If $|r| \geq 1$ then the series diverges.

Example: The sum of the geometric sequence $a_n = 3 * (\frac{1}{2})^{n-1}$ il

$$\sum_{n=1}^{\infty} a_n = \frac{3}{1 - (\frac{1}{2})} = 6$$

Exercises:

1) Does the series $4 + \frac{4}{5} + \frac{4}{25} + \frac{4}{125} + \cdots$ converge? If so, what is the sum?
2) What about $4 + 12 + 36 + \cdots$
3) A pendulum swings through an arc 40 cm long on its first swing. Each swing reduces the length of the swing by 20 percent. How far will it swing before coming to a stop?