Section 3/17: 6.6, 9.5, 12.3

Tuesday, March 17, 2020 9:04 AM

Differential: The **differential** df of a single variable function f(x) for a very small number dx is the expression

$$df = f'(x)dx$$

The differential is a helpful tool for approximating values of a function f near values that you already know.

Linear Approximation: Suppose that f is a function of x and dx is a very small number. Then

$$f(x + dx) \approx f(x) + df = f(x) + f'(x)dx$$

So f(x + dx) is **approximately** equal to the right-hand side.

Example (Small Angle): Suppose that f(θ) = sin(θ) and dθ is a small amount of angle (so close to 0). Then

sin($d\theta$) = sin(0 + $d\theta$) $\approx \sin'(0)d\theta$ = $d\theta$

So for small angles, sine of an angle is approximately just the angle itself. This is called the **small angle approximation** in physics.

Evaluating: If I give you a function f(x), a value x and a small value dx, I can ask you to evaluate the differential, meaning find the value f'(x) * dx.

Exercise 1. Approximate $\sqrt[3]{126}$ using differentials, and then use a calculator to find the error up to 4 decimal places.

Exercise 2. The radius of a blood vessel is 1.7 mm. A drug causes the radius to contract to 1.6 mm. Find the approximate change in the area of the cross section.

Multivariable Differentials: Multivariable functions also have a notion of differential.

The **total differential** df of a two variable function f(x,y) at x and y is

$$df = f_x(x,y)dx + f_y(x,y)dy$$

Linear Approximation: The same basic linear approximation fact is still true. When x,y are some numbers and dx, dy are teeny tiny numbers, then

$$f(x+dx, y+dy) \approx f(x,y) + df = f_x(x,y)dx + f_y(x,y)dy$$

Exercise 3. Evaluate dz given the following information.

$$z = \ln(x^2 + y^2) \quad x = 2, y = 3 \quad dx = .02, dy = -.03$$

Exercise 4. A piece of bone in the shape of circular cylinder is 7 cm long and has a radius of 1.4 cm. You coat the bone with a layer of preservative that is .09 cm thick. Estimate the volume of preservative used.

Taylor Polynomials. There is a version this approximation process that uses more than just the first derivative.

The **n-th order Taylor polynomial** of a function f(x) is the polynomial.

$$P(x) = \sum_{i=0}^{n} \frac{f^{(i)}(0)}{i!} x^i$$