Worksheet 5: 8.2, 9.6

Exercise 1 (§9.6 # 1,9) Evaluate each integral

$$\int_0^5 (x^4y+y)dx \qquad \text{and} \qquad \int_0^3 ye^{4x+y^2}dy$$

$$= \left(\frac{1}{5}x^5y + xy\right)\Big|_0^5$$

$$= \boxed{5^4\cdot y + 5\cdot y}$$

$$= e^{4x}\int_0^3 ye^{y^2}dy$$

$$= e^{4x}\int_0^3 \left(\frac{1}{2}e^{y^2}\right)' dy$$

$$= e^{4x}\left(\frac{1}{2}e^{y^2}\Big|_0^3\right)$$

$$= e^{4x}\left(\frac{1}{2}e^9 - \frac{1}{2}\right)$$

$$= \boxed{\frac{1}{2}e^{4x}(e^9-1)}$$

Exercise 2 (§9.6, # 23) Evaluate each double integral.

$$\int_1^5\int_0^3 (x^2y+5y)dxdy \qquad \text{and} \qquad \int\int_R \sqrt{x+y}\,dydx; \quad 1\le x\le 3, 0\le y\le 1$$

$$= \int_1^5 \left(\frac{1}{3}x^3y + 5yx\right)\Big|_0^3 dy$$

$$= \int_1^5 \frac{1}{3}3^3y + 15y\ dy$$

$$= \int_1^5 24y\ dy$$

$$= 12y^2\Big|_1^5 = 12\cdot(25-1)$$

$$= \boxed{288}$$

$$\int_1^3\int_0^1 \sqrt{x+y}\ dy\,dx$$

$$= \int_1^3 \frac{2}{3}(x+y)^{3/2}\Big|_{y=0}^{y=1} dx$$

$$= \frac{2}{3}\int_1^3 (x+1)^{3/2} - x^{3/2}\,dx$$

$$= \frac{2}{3}\left(\frac{2}{5}(x+1)^{5/2} - \frac{2}{5}x^{5/2}\right)\Big|_{x=1}^{x=3}$$

$$= \frac{4}{15}\left((4^{5/2} - 3^{5/2}) - (2^{5/2} - 1)\right)$$

$$= \boxed{\frac{4}{15}\left(2^5+1 - 3^{5/2} - 2^{5/2}\right)}$$

Exercise 3 (§9.6, # 39, 45) Evaluate each double integral.

$$\int_2^4 \int_2^{x^2} (x^2 + y^2)dydx \qquad \text{and} \qquad \int_1^4 \int_1^{e^x} \frac{x}{y}dydx$$

$= \int_2^4 (x^2y + \frac{1}{3}y^3)\Big|_{y=2}^{y=x^2} dx$

$= \int_2^4 x^4 + \frac{1}{3}x^6 - 2x^2 - \frac{1}{3}8 \, dx$

$= (\frac{1}{21}x^7 + \frac{1}{5}x^5 - \frac{2}{3}x^3 - \frac{8}{3}x)\Big|_2^4$

$= \boxed{929.83}$ approx

$\int_1^4 x\ln(y)\Big|_{1=y}^{e^x=y} dx$

$= \int_1^4 x(\ln(e^x) - \ln(1)) dx$

$= \int_1^4 x^2 dx = \frac{1}{3}x^3\Big|_1^4$

$= \boxed{\frac{2^6-1}{3}}$

Execise 4 (§8.2, # 39) The ~~average~~ yearly corn production in the US (in billions of bushels) was approximately given by

$$p(t) = 1.757(1.0248)^{t-1930}$$

between 1930 and 2010. Find the average yearly corn production from 1930 to 1950, and the average from 2000 to 2010. Use a calculator to get an answer with 4 significant digits.

Note that

$1.0248^{t-1930} = e^{\ln(1.0248)(t-1930)}$

$\Rightarrow \int p(t)\,dt = \frac{1.757}{\ln(1.0248)} \cdot e^{\ln(1.0248)(t-1930)}$

$p(t) = 1.757e^{\ln(1.0248)(t-1930)}$

avg over a to b $= \frac{\int_a^b p(t)\,dt}{b-a}$

avg 1930 to 1950

$\frac{\frac{1.757}{\ln(1.0248)}\left(e^{\ln(1.0248)(t-1930)}\right)\Big|_{1930}^{1950}}{1950-1930}$

$= \boxed{2.267}$ billion bushels/year

avg 2000 to 2010

$\frac{\frac{1.757}{\ln(1.0248)}\left(1.0248^{(t-1930)}\right)\Big|_{2000}^{2010}}{2010-2000} =$

$= \boxed{11.06}$ billions bushels/year