Worksheet 1: 13.1-13.3

Exercise 1 Take the following derivatives.

$$\cos(x^2) \qquad \frac{\cos(2x)}{2+\cos(x)} \qquad \text{and} \qquad \frac{\cos^2(x)}{1-\sin^2(x)}$$

(i) chain rule.

$\cos(x^2)' = -\sin(x^2)\cdot 2x = \boxed{-2x\sin(x^2)}$

(ii) quotient rule

$$\left(\frac{\cos(2x)}{2+\cos(x)}\right)' = \frac{\cos(2x)'\cdot(2+\cos(x)) - \cos(2x)\cdot(2+\cos(x))'}{(2+\cos(x))^2} =$$

$$\frac{-2\sin(2x)\cdot(2+\cos(x)) - \cos(2x)\cdot(-\sin(x))}{(2+\cos(x))^2} = \boxed{\frac{-4\sin(2x) + \cos(2x)\sin(x) - 2\sin(2x)\cos(x)}{(2+\cos(x))^2}}$$

(iii) $1 = \cos^2(x) + \sin^2(x)$ trig Id $\Rightarrow \dfrac{\cos^2(x)}{1-\sin^2(x)} = \dfrac{\cos^2(x)}{\cos^2(x)} = 1 \Rightarrow (1)' = \boxed{0}$.

Exercise 2 (§13.3 # 21) Take the following indefinite integrals.

$$\int e^x \sin(e^x)dx \qquad \int -6x\sin(x)dx \quad \text{and} \quad \int \frac{\sin(x)}{\sqrt{\cos(x)}}dx$$

(i) u-substitution. $u = e^x$, $du = e^x dx$

$\int e^x \sin(e^x)\,dx = \int \sin(u)\,du = \cos(u) + C = \boxed{\cos(e^x) + C.}$

(ii) integration by parts. $u = -6x$, $du = -6dx$; $v = -\cos(x)$, $dv = \sin(x)dx$ } LATE rule applies $u =$ Algebraic $= -6x$

$\int -6x\cdot\sin(x) = -6x\cdot -\cos(x) - \int -6\cdot -\cos(x)\,dx$

$= \boxed{6x\cos(x) - 6\sin(x) + C}$

(iii) u-substitution. $u = \cos(x)$ $du = -\sin(x)$

$\int \frac{\sin(x)}{\sqrt{\cos(x)}}dx = -\int \frac{1}{\sqrt{u}}du = -2\sqrt{u} + C = \boxed{-2\sqrt{\cos(x)} + C}$

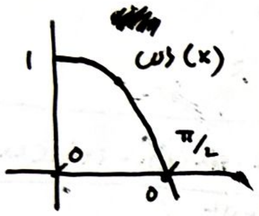
Exercise 3 (§8.1 # 35) Find the following integral

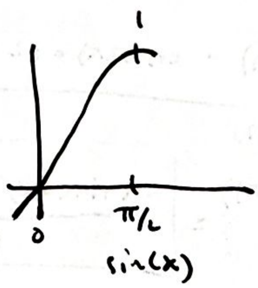
$$\int_0^{\pi/2} \sqrt{1 + 2\cos(x)\sin(x)}dx$$

trig identity

$1 = \cos^2(x) + \sin^2(x)$

$\Rightarrow \sqrt{1 + 2\cos(x)\sin(x)} = \sqrt{(\cos(x) + \sin(x))^2} = |\cos(x) + \sin(x)|$





$\Rightarrow |\cos(x) + \sin(x)|$

$= \cos(x) + \sin(x)$

for $0 \le x \le \frac{\pi}{2}$

$\Rightarrow \int_0^{\pi/2} \sqrt{1 + 2\cos(x)\sin(x)}\,dx = \int_0^{\pi/2} \cos(x) + \sin(x)\,dx$

$= \sin(x) - \cos(x) \Big|_0^{\pi/2}$

$= \left(\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)\right)$

$-\left(\sin(0) - \cos(0)\right)$

$= \boxed{2}$