Worksheet 3: 9.1-9.2

Exercise 1 (§9.1 # 5) Let $f(x, y) = e^x + \ln(x + y)$. Find the values of f at $(x, y) = (1, 0)$, $(2, -1)$, $(0, e)$ and $(0, e^2)$.

- $f(1,0) = e^1 + \ln(1+0) = e + 0 = \boxed{e}$
- $f(2,-1) = e^2 + \ln(1) = e^2 + 0 = \boxed{e^2}$
- $f(0,e) = e^0 + \ln(e) = 1 + 1 = \boxed{2}$
- $f(0,e^2) = e^0 + \ln(e^2) = 1 + 2 = \boxed{3}$

Exercise 2 (§9.1 # 29) Let $f(x, y) = 4x - 2y^2$ and find the following

$$\frac{f(x+h, y) - f(x, y)}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

- $$\frac{f(x+h,y) - f(x,y)}{h} = \frac{(4(x+h) - 2y^2) - (4x - 2y^2)}{h}$$

$$= \frac{4h + \cancel{4x} - \cancel{2y^2} - \cancel{4x} + \cancel{2y^2}}{h} = \boxed{4}$$

- $\lim_{h \to 0} 4 = \boxed{4}$

Exercise 3 (§9.2 # 13, 15, 17) Take the partial derivatives f_x and f_y for the following functions of x and y.

(i) $\ln|1+5x^3y^2|$ (ii) $\sqrt{x^4+3xy+y^4+10}$ (iii) $\frac{3x^2y}{e^{xy}+2}$

(i) $f_x = \frac{15x^2y^2}{|1+5x^3y^2|}$ (chain rule) $f_y = \frac{10x^3y}{|1+5x^3y^2|}$

(ii) $f_x = \frac{1}{2}(x^4+3xy+y^4+10)^{-1/2} \cdot (4x^3+3y)$ (chain rule)

$f_y = \frac{1}{2}(x^4+3xy+y^4+10)^{-1/2} \cdot (3x+4y^3)$

(iii) $f_x = \frac{(e^{xy}+2)\cdot 6xy - ye^{xy}\cdot 3x^2y}{(e^{xy}+2)^2}$

$f_y = \frac{(e^{xy}+2)\cdot 3x^2 - xe^{xy}\cdot 3x^2y}{(e^{xy}+2)^2}$

Exercise 4 (§9.2 # 37) Find f_x, f_y and f_z for $f(x,y)$ given by

$$2x^2+3xy-4z^5$$

$f_x = 4x+3y$

$f_y = 3x$

$f_z = 20z^4$