Worksheet 1/28 Solutions

#1 (§8.1 #9)

$$u = \ln(3x) \qquad v = x$$

$\int_1^9 \ln(3x)\,dx$. Integration by parts w/ $du = \frac{1}{x}\,dx \qquad dv = 1 \cdot dx$

$$\Rightarrow \int_1^9 \ln(3x)\,dx = \ln(3x)\cdot x \Big|_1^9 - \int_1^9 x \cdot \frac{1}{x}\,dx$$

$$= \ln(3x)\cdot x \Big|_1^9 - x\Big|_1^9$$

$$= \left(\ln(3\cdot 9)\cdot 9 - \ln(3)\cdot 1\right) - (9-1)$$

$$= \boxed{26 \cdot \ln(3) - 8}$$

Note:
$\ln(27) = \ln(3^3)$
$= 3 \cdot \ln(3)$

#2 (§8.1 #21) $\int_0^1 \frac{x^3}{\sqrt{3+x^2}}\,dx$

1st way: u substitution.

$$u = 3 + x^2 \qquad du = 2x\,dx \Rightarrow dx = \frac{1}{2x}\,du$$

$$\int_0^1 \frac{x^3}{\sqrt{3+x^2}}\,dx = \int_0^1 \frac{x^3}{\sqrt{u}} \cdot \frac{1}{2x}\,du = \frac{1}{2}\int_3^4 \frac{x^2}{\sqrt{u}}\,du = \frac{1}{2}\int_3^4 \frac{u-3}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int_3^4 \sqrt{u} - \frac{3}{\sqrt{u}}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} - 6u^{1/2}\right)\Big|_3^4 = \frac{1}{2}\left(\frac{2}{3}\cdot 2^3 - 6\cdot 2\right) - \frac{1}{2}\left(\frac{2}{3}\cdot 3^{3/2} - 6\cdot 3^{1/2}\right)$$

$$\boxed{= .1307}$$

2nd way: integration by parts:

$$\int_0^1 \frac{x^3}{\sqrt{3+x^2}}\,dx \rightsquigarrow \quad u = x^2 \qquad v = \sqrt{3+x^2}$$

$$du = 2x\,dx \qquad dv = \frac{x}{\sqrt{3+x^2}}\,dx$$

$$\int_0^1 \frac{x^3}{\sqrt{3+x^2}}\,dx = x^2\sqrt{3+x^2}\,\Big|_0^1 - \int_0^1 2x\sqrt{3+x^2}\,dx$$

derivative

$$= x^2\sqrt{3+x^2}\,\Big|_0^1 - \int_0^1 \left(\tfrac{2}{3}(3+x^2)^{3/2}\right)'dx$$

$$= x^2\sqrt{3+x^2}\,\Big|_0^1 - \tfrac{2}{3}(3+x^2)^{3/2}\,\Big|_0^1$$

$$= (\sqrt{4} - \sqrt{0}) - \left(\tfrac{2}{3}4^{3/2} - \tfrac{2}{3}3^{3/2}\right) = \boxed{.1307}$$

\#3 (§ 8.1 #35)

integration by parts...

$$\int x^n \cdot \ln|x|\,dx \qquad u = \ln|x| \qquad v = \frac{1}{n+1}x^{n+1}$$

$$du = \frac{1}{x}\,dx \qquad dv = x^n$$

$$\int x^n \ln|x|\,dx = \frac{1}{n+1}x^{n+1}\ln|x| - \int \frac{1}{n+1}x^n\,dx$$

$$= \frac{1}{n+1}x^{n+1}\cdot\ln|x| - \frac{1}{(n+1)^2}\cdot x^{n+1} + C$$

$$= \boxed{x^{n+1}\cdot\left(\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right) + C}$$

#4 (§ 8.1 #41). The problem is asking for

$$\int_0^2 (27t)\cdot e^{3t}\,dt \Rightarrow \quad u = 27t \quad v = \frac{1}{3}e^{3t}$$
$$du = 27\,dt \quad dv = e^{3t}\,dt$$

$$\int_0^2 (27t)\cdot e^{3t} = \frac{27t}{3}\cdot e^{3t}\Big|_0^2 - \int_0^2 \frac{27}{3}e^{3t}\,dt = \int_0^2 \left(\frac{27}{3^2}\cdot e^{3t}\right)'dt = \frac{27}{3^2}\cdot e^{3t}\Big|_0^2$$

$$\boxed{= \left(9te^{3t} - 3e^{3t}\right)\Big|_0^2 = 6054.43..}$$

#5 (§ 8.3 #11) Let's denote

P = "present value" $\quad A$ = "accumulated money"

then (by book formulas)

$$P = \int_0^{10} F(t)\,e^{-rt}\,dt = \int_0^{10} (.02t + 300)e^{-.08t}\,dt$$
$$= \int_0^{10}\left(\frac{2}{100}t + 300\right)e^{\frac{-8t}{100}}\,dt$$

integration by parts

$$u = \frac{2}{100}t + 300 \qquad v = \frac{-100}{8}e^{-8/100\,t}$$
$$du = \frac{2}{100}\,dt \qquad dv = e^{-8t/100}\,dt$$

$$\Rightarrow P = \left(\frac{2}{100}t + 300\right)\cdot\frac{-100}{8}e^{\frac{-8t}{100}}\Big|_0^{10} - \int_0^{10} -\frac{2}{8}e^{-8/100\,t}\,dt$$
$$= \left(\frac{2}{100}t + 300\right)\cdot -\frac{100}{8}e^{-8t/100}\Big|_0^{10} - \left(-\frac{200}{8^2}e^{-8/100\,t}\right)\Big|_0^{10}$$

$$\boxed{= \left(\frac{-2}{8}t - \frac{3\cdot10^4}{8} + \frac{200}{8^2}\right)e^{-8t/100}\Big|_0^{10} = 2062.17}$$

#6 (§ 8.2 #17)

Need to find $F(t)$...

$$F(t) = \underbrace{5000}_{\text{initial \$}} \cdot e^{-.01 \cdot t}$$

↑ exponentially at .01 per year decrease. (compounded continuously).

$r = 8\% = .08$

$$P = \int_0^8 F(t)\, e^{-rt}\, dt = \int_0^8 5000 \cdot e^{-.01 \cdot t} \cdot e^{-.08 \cdot t}\, dt$$

$$= \int_0^8 5000 \cdot e^{-.09t}\, dt = \left(5000 \cdot \frac{-1}{.09} \cdot e^{-.09t}\right)\Big|_0^{10}$$

$$\boxed{= \$32968.35 \ldots}$$