Solving Homogeneous Differential Equations

Problem 1. *Find the general solution to the following ODEs:*

1. $y'' + 3y = 0$

2. $y'' - y' + 9y = 0$

3. $y''' + y' - 2y = 0$

Method of Undetermined Coefficients

Problem 2. *Find the particular solution to the following ODEs:*

1. $y'' + 3y = -9$

2. $y'' - y' + 9y = 3\sin 3t$

3. $y''' + y' - 2y = te^t$

Problem 3. *Write the general solution to the above differential equations.*

Variation of Parameters

Problem 4. *Use variation of parameters to determine the particular solution to one of the problems in problem 2.*

Problem 5. *Find a general solution to* $\dfrac{1}{2}y'' + 2y = \tan 2t - \dfrac{1}{2}e^t$

Problem 6. *Find a general solution the differential equation* $ty'' - (t+1)y' + y = t^2 e^{2t}$ *given that* $f(t) = e^t$ *is a homogeneous solution [**hint:** use reduction of order].*

System of Differential Equations

Problem 7. *Write:*

$$\begin{cases} r'(t) = 2r(t) + \sin t \\ \theta'(t) = r(t) - \theta(t) + 1 \end{cases}$$

in the form $\vec{x}' = A\vec{x} + \vec{f}$

Problem 8. *Find a general solution to* $x' = Ax$ *for* $A = \begin{pmatrix} 1 & 3 \\ 12 & 1 \end{pmatrix}$

Problem 9. *Find a general solution to* $x' = Ax$ *for* $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

Problem 10. *Write* $x'' + x = 0$ *in the form* $\vec{y}' = A\vec{y}$ *and use techniques to solve this differential equation.*