(4.3, 4.4, 4.5 Auxiliary Equations with complex roots, method of undetermined coefficients, principal of superposition)

Problem 1. *Solve the initial value problem:*

$$\begin{cases} y'' - 2y' + 7y = 0 \\ y(0) = 1 \\ y'(0) = -2 \end{cases}$$

Problem 2. *Solve the initial value problem:*

$$\begin{cases} y'' + by' + 4y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

for $b = 5, 4, 2$ and sketch to solutions. Explain what these mean if this equation was modeling a mass attached to a spring.

Method of Undetermined Coefficients

To find a particular solution to the differential equation
$$ay'' + by' + cy = Ct^m e^{rt} \ ,$$
where m is a nonnegative integer, use the form

(14) $y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{rt} \ ,$

with

(i) $s = 0$ if r is not a root of the associated auxiliary equation;
(ii) $s = 1$ if r is a simple root of the associated auxiliary equation; and
(iii) $s = 2$ if r is a double root of the associated auxiliary equation.

To find a particular solution to the differential equation
$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t}\cos \beta t \\ \quad \text{or} \\ Ct^m e^{\alpha t}\sin \beta t \end{cases}$$
for $\beta \neq 0$, use the form

(15) $y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t} \cos \beta t$
            $+ t^s(B_m t^m + \cdots + B_1 t + B_0)e^{\alpha t} \sin \beta t \ ,$

with

(iv) $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and
 (v) $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.

Problem 3. *Find a particular solution to*

1. $y'' + 2y' - y = 10$

2. $2x' + x = 3t^2$

3. $y''(\theta) - y'(\theta) = \theta^2$

4. $y'' + 2y' + 4y = 111e^{2t}\cos(3t)$

Problem 4. *Find a general solution to*

$$y'' - y = -11t + 1$$

Problem 5. *Find the solution to the initial value problem*

$$\begin{cases} y'' + 9y = 27 \\ y(0) = 4 \\ y'(0) = 6 \end{cases}$$