Math 54 – Worksheet 5
GSI: Oltman, (10/17/19)

(5.1,5.2,5.3,5.4) Eigenvalues, Eigenvectors, Characteristic equation, Diagonalization, Geometry of Diagonalization

Problem 1. *Compute the eigenvalues and eigenvectors for* $A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$

Problem 2. *Determine the eigenvalues for:*

$$A = \begin{pmatrix} 186 & 324 \\ 117 & 303 \end{pmatrix}$$

this problem is courtesy of my officemate Adam.

Problem 3. *Compute* D^6 *for* $D = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$

Problem 4. *Compute* A^6 *for* $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$*, given the information that*

$$A = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

Problem 5. *What are the eigenvalues and eigenvectors for the above matrix?*

Problem 6. *Compute the eigenvalues and eigenvectors for:*

$$A = \begin{pmatrix} 4 & 0 & 2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix}$$

Problem 7. *Define* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *by* $T(x) = \begin{pmatrix} 0 & 1 \\ -3 & 4 \end{pmatrix}$*. Find a basis* $\mathcal{B}$ *such that* $[T]_{\mathcal{B}}$ *is diagonal.*

Problem 8. *Determine if the following are true or false*

1. *A matrix can have* 0 *as an eigenvalue*
2. *A matrix can have* $\vec{0}$ *as an eigenvector*
3. *If* A, B, C *are square matrices such that* $A = BCB^{-1}$ *then* A *and* C *have the same eigenvalues*
4. *If* A, B, C *are square matrices such that* $A = BCB^{-1}$ *then* A *and* C *have the same eigenvectors*

Problem 9. *Determine all possible eigenvalues of* A *if* $A^2 = 1$*.*