Math 54 – Worksheet 4

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Problem 1. *Verify by hand the the inverse of the matrix* $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *is*

$$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Solution 1. By computation:

$$A^{-1}A = \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = \mathbb{I}_2$$

And:

$$AA^{-1} = \mathbb{I}_2$$

Problem 2. *Find the inverse (if it exists) of*

$$\begin{pmatrix} 1 & -2 & -1 \\ 0 & 3 & 5 \\ 0 & 6 & 10 \end{pmatrix}$$

Problem 3. *This matrix is row equivalent to:*

$$\begin{pmatrix} 1 & -2 & -1 \\ 0 & 1 & 5/3 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore it is not one-to-one, therefore it has no inverse.

Problem 4. *Find the inverse (if it exists) of*

$$\begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

Solution 2. By the algorithm, we have:

$$\left(\begin{array}{cccccc} 10 & -2 & 1 & 0 & 0 & \\ -3 & 1 & 4 & 0 & 1 & \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow{RREF} \begin{pmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{pmatrix}$$

Therefore the inverse is:

$$\begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{pmatrix}$$

Problem 5. *Find the inverse (if it exists) of*

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Solution 3. The inverse is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Problem 6. *Find the inverse of* $A = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$ *and* $B = \begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix}$. *Then compute the inverse of their product:* AB.

Solution 4. This problem is computation, but is trying to get you to see that $(AB)^{-1} = B^{-1}A^{-1}$

These problems don't have to do with inverses, but you need to know how to solve them

Problem 7. *Consider the set of all real values* x, y, z, w *such that* $2x - 2z + w = 0$ *and* $y - w = 0$. *Call this set* S.

1. *Find a matrix* A *such that* $S = null(A)$.
2. *Determine the dimension of the kernel of* A *and the dimension of the column space of* A *(also known as the rank of* A*).*

Solution 5. 1.

$$A = \begin{pmatrix} 2 & 0 & -2 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

2. We can compute the Kernel (which is another word for the null space) by row reducing:

$$\begin{pmatrix} 2 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 0 & -1/2 & 1/2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

Therefore the solutions are:

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = t \begin{pmatrix} 1/2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1/2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

These two vectors are your basis for the Kernel, therefore the kernel has two dimensions. A has two pivot columns, therefore the dimension of the column space is 2.

Problem 8. *Find a basis for* $\mathbb{R}^4$, *that contains the following vectors:*

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 0 \\ 3 \end{pmatrix} \right\}$$

Solution 6. Put four more vectors in that you know span $\mathbb{R}^4$, then reduce to a linearly independent one.

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Put this in a big matrix and row reduce it to get:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

There is a pivot in the first, second, third, and fourth columns. Therefore we get that the first, second, third, and fourth columns of the original matrix are linearly independent, therefore we have that the following is a basis for $\mathbb{R}^4$:

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$