Math 54 – Worksheet 3
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Problem 1. *Let* $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $v_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$, *and* $v_2 = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$. *And let* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *be a linear transformation that maps* x *into* $x_1 v_1 + x_2 v_2$. *Find a matrix* A *such that* $T(x) = Ax$ *for each* x.

Problem 2. *Show that the transformation* T *defined by* $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ *is not linear.*

Problem 3. *In this problem we will consider the space of polynomials (up to degree 3) as a vector space. We will identify the arbitrary polynomial* $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ *with the vector* $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$.

1. *Write the constant polynomial* $p(x) = 1$ *as a vector in this way.*
2. *Compute the derivative of* $p(x)$ *and write this new polynomial as a vector*
3. *Repeat this process for the polynomials* $p(x) = x, x^2, x^3$.
4. *Put all the vectors you found in step* 2 *into a* 4×4 *matrix,* D. *Verify the following claim*

   Claim 1. *Given an arbitrary polynomial of degree at most* 4. *If you first convert this into a vector, apply* D *to it, then convert it back into a polynomial, you get the derivative of your original polynomial.*

5. *think about this*

Problem 4. *Let* $\vec{v}_1 = (4, 5)$ *and* $\vec{v}_2 = (1, 3)$ *and suppose that* T *is a linear map that sends* $\vec{v}_1$ *to* $\vec{v}_2$ *and* $\vec{v}_2$ *to* $\vec{v}_1$. *Find* $T((1, 1))$.