Math 54 Worksheet

GSI: Izak Oltman

September 13, 2019

Solve the following problems (and let me know if you have any questions or catch a typo).

1. Is $(1, 2, 0)$ in the span of $(1, 0, 0), (0, 1, 0)$? **Solution:**

   Yes, because:

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

2. Suppose I throw a ball from the position $\vec{x}_0$ with velocity vector $\vec{v}(t)$ such that:

$$\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{v}(t) = \begin{pmatrix} 1 \\ 2 \\ 1 - 10t \end{pmatrix}$$

   Determine the location of the ball when the third coordinate of the position vector of the ball is zero.
   Solution:

   You don't need to know how to do this, I thought it would be a fun physics problem, but you don't need to know physics for this course.

   Integrate the velocity vector and use the initial position to determine the position vector:

$$\vec{x} = \begin{pmatrix} t \\ 2t \\ t - 5t^2 + 1 \end{pmatrix}$$

   Solve for the time when the z coordinate is 0, that is $t - 5t^2 + 1 = 0$ so $t_0 = \dfrac{-5 + \sqrt{21}}{10}$ (or something). Then compute $\vec{x}(t_0)$ and you get your position.

3. Write the vector $(9, 6)$ as a linear combination of the vectors $(1, 2)$ and $(1, -4)$

Solution:

This is equivalent to row reducing the augmented matrix:

$$\begin{pmatrix} 1 & 1 & 9 \\ 2 & -4 & 6 \end{pmatrix}$$

4. Construct a 3×3 matrix A and vectors $b, c \in \mathbb{R}^3$ so that $Ax = b$ has a solution but $Ax = c$ does not.

   Solution:

   The trick is to make some matrix A such that b is in the column space but c is not. A trivial example would be:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

with $b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $c = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

5. Write down the following linear system of equations in the form $Ax = b$

$$\begin{aligned} x - 3y + 4z &= -4 \\ 3x - 7y + 7z &= -8 \\ -4x + 6y - z &= 7 \end{aligned}$$

   Solution:

$$\begin{pmatrix} 1 & -3 & 4 \\ 3 & -7 & 7 \\ -4 & 6 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ 7 \end{pmatrix}$$

6. Let $\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$ and $A = \begin{pmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{pmatrix}$ is $\vec{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

   Solution:

   This is equivalent to row reducing the following matrix and seeing if there exists a solution:

$$\begin{pmatrix} 5 & 8 & 7 & 2 \\ -3 & 0 & 1 & -1 \\ 2 & 1 & 3 & 0 \end{pmatrix}$$

   It turns out that there is a solution, therefore $\vec{u}$ is in the span of the columns of A