

Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name: _____

1. (3pts)

- (a) Find the general solution to $y'' - y' - 6y = 0$
- (b) Find the solution to $y'' - y' - 6y = 0$ subject to the initial conditions: $y(0) = 1$, $y'(0) = 1$
- (c) Suppose that y is a function of time that solves the initial value problem in part (b), determine the long term behavior of y

- (a) the auxillary equation is $r^2 - r - 6$ so $(r - 3)(r + 2) = 0$ so $r = 3, -2$. Therefore $y = ae^{3t} + be^{-2t}$
- (b) We have $1 = a + b$ and $1 = 3a - 2b$. Therefore: $b = 2/5$ and $a = 3/5$.
- (c) y goes to infinity.

2. (2pts) Find the general solution to:

$$y''' + 2y'' + 5y' - 26y = 0$$

We have $r^3 + 2r^2 + 5r - 26 = 0$. $r = 2$ solves this, then we can preform polynomial long division to get the factor $(r - 2)(r^2 + 4r + 13)$. We then get imaginary roots $-2 \pm 3i$. Therefore the general solution is:

$$y(t) = c_1 e^{2t} + c_2 e^{-2t} \cos(3t) + c_3 e^{-2t} \sin(3t)$$

3. (2pt) Find the general solution to:

$$y'' - 2y' + y = t^2 - 1$$

The homogeneous solution is found by solving $r^2 - 2r + 1$ so $r = 1$ repeated, so we have $y = c_1 e^t + c_2 t e^t$. The particular solution is found by guessing $y_p = at^2 + bt + c$. We then must solve:

$$t^2 - 1 = 2a - 4at - 2b + at^2 + bt + c$$

Therefore $a = 1$, $-4a + b = 0$ so $b = 4$, $-1 = 2a - 2b + c = 2 - 8 + c$, therefore $c = 5$. So our general solution is:

$$c_1 e^t + c_2 t e^t + t^2 + 4t + 5$$