

Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name: \_\_\_\_\_

1. (2pts) Compute the orthogonal projection of  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  onto the line through  $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

If  $u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  and  $v = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$  then this is just:

$$\frac{u \cdot v}{v \cdot v} v = \frac{-1 - 3}{1 + 9} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \frac{-2}{5} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

2. (3pts) Let  $y = \begin{pmatrix} 5 \\ -9 \\ 5 \end{pmatrix}$ ,  $u_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}$ ,  $u_2 = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$ . Find the distance from  $y$  to the plane in  $\mathbb{R}^3$  spanned by  $u_1$  and  $u_2$

$u_1$  and  $u_2$  are orthogonal, therefore we just need to compute  $\|y - \hat{y}\|$ , where  $\hat{y} = \text{proj}_{u_1} y + \text{proj}_{u_2} y$ . Therefore:

$$\begin{aligned} \hat{y} &= \frac{-15 + 45 + 5}{9 + 25 + 1} \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix} + \frac{-15 - 18 + 5}{9 + 4 + 1} \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = \frac{35}{35} \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix} + \frac{-28}{14} \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -3 + 6 \\ -5 - 4 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ -1 \end{pmatrix} \end{aligned}$$

Therefore:

$$\|y - \hat{y}\| = \left\| \begin{pmatrix} 5 - 3 \\ -9 + 9 \\ 5 + 1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix} \right\| = \sqrt{40}$$

3. Draw a spooky Halloween cartoon involving linear algebra on the back of this quiz.