Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name:____

1. (5 pts) Fill in the blank with the most correct answer

   (a) The 3×3 identity matrix has eigenvalue 1 with algebraic multiplicity ______ and geometric multiplicty ______

   (b) For each eigenvalue of a matrix, the algebraic multiplicity is always ______ the geometric multiplicity ($<, \leq, >, \geq$)

   (c) The sum of the eigevenvalues of a matrix A equals the ________________ of A, and the product of the eigenvalues equals the ________________ of A

   (a) 3, 3

   (b) $\geq$

   (c) *trace*, det

2. (2pts) Given a matrix A with eigenvalues -1 and 2 with eigenvectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ respectively. Compute A^4 (you don't need to multiply every scalar out, for example you can leave $14 \cdot 16 \cdot 5$ in your answer)

   Let $P = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ then $P^{-1} = -\begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$. Therefore we have that:

   $$A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$$

   Therefore:

   $$A^4 = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2^4 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$$

3. (5pts) Compute the following things for a matrix M. [**hint:** this problem involves complex numbers]

   (a) (2pts) Compute the eigenvalues (λ_1, λ_2) and eigenvectors $(\vec{w}_1, \vec{w}_2)$ for $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

   (b) (2pt) Write $\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ as a linear combination of the eigenvectors of M ($\vec{v} = a\vec{w}_1 + b\vec{w}_2$)

   (c) (1pt) Compute $M^2\vec{v}$ by computing $a^2\vec{w}_1 + b^2\vec{w}_2$

   (a) The characteristic polynomial is $\lambda^2 + 1 = 0$ therefore $\lambda_1 = i$, $\lambda_2 = -i$.For λ_1 we have the eigenvector from the null space of :

   $$\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}$$

so the eigenvector is:

$$w_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}$$

Therefore for λ_2 we have:

$$w_2 = \begin{pmatrix} -i \\ 1 \end{pmatrix}$$

(b) We need to row reduce the augmented matrix:

$$\begin{pmatrix} i & -i & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

We get that $a = 3/2 - i$ and $b = 3/2 + i$.

(c) There was a typo on this problem, it meant for you to compute $a\lambda_1^2\vec{w}_1 + b\lambda_2^2\vec{w}_2$ which is equal to $M^2\vec{v}$. This computation doesn't make any sense. For this reason, I gave everyone a point for this problem regardless, and gave an extra credit point if anyone caught the typo.

The End.