MATH 54 – DISC 203 QUIZ 7 OCTOBER 24TH, 2019

Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name:_____

1. (5 pts) Fill in the blank with the most correct answer
   (a) The 3×3 identity matrix has eigenvalue 1 with algebraic multiplicity _____ and geometric multiplicty _____
   (b) For each eigenvalue of a matrix, the algebraic multiplicity is always _____ the geometric multiplicity $(<, \leq, >, \geq)$
   (c) The sum of the eigevenvalues of a matrix A equals the _____ of A, and the product of the eigenvalues equals the _____ of A

2. (2pts) Given a matrix A with eigenvalues -1 and 2 with eigenvectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ respectively. Compute A^4 (you don't need to multiply every scalar out, for example you can leave $14 \cdot 16 \cdot 5$ in your answers)

3. (5pts) Compute the following things for a matrix M. [**hint:** this problem involves complex numbers]
   (a) (2pts) Compute the eigenvalues (λ_1, λ_2) and eigenvectors $(\vec{w}_1, \vec{w}_2)$ for $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
   (b) (2pt) Write $\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ as a linear combination of the eigenvectors of M $(\vec{v} = a\vec{w}_1 + b\vec{w}_2)$
   (c) (1pt) Compute $M^2\vec{v}$ by computing $a^2\vec{w}_1 + b^2\vec{w}_2$