MATH 54 – DISC 203 QUIZ 6 OCTOBER 17TH, 2019

Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name:____

1. (4pts) Write your answer in the blank:

   (a) (T/F) If $\vec{v}$ is an eigenvector with eigenvalue λ for a matrix A, then $c\vec{v}$ (with $0 \neq c \in \mathbb{R}$) is an eigenvector with eigenvalue $c\lambda$. ______

   (b) If an invertible matrix A has an eigenvector $\vec{v}$ with eigenvalue λ, then A^{-1} has eigenvector ______ with eigenvalue ______

   (c) (T/F) If A is a matrix with eigenvalue 0 then A is invetible ______

   (a) False

   (b) v, λ^{-1}

   (c) False

2. (4pts) Determine the eigenvalues and eigenvectors for the following matrix:

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$$

The characteristic polynomial is $0 = (2-\lambda)(4-\lambda)+1 = \lambda^2 - 6\lambda + 9 = (\lambda-3)^2$. So we have $\lambda = 3$ with algebraic multiplicity 2. We can compute the eigenvector by finding a basis for the null space of:

$$\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$$

which is just $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

3. (2pt) If A (in the previous problem) acts on the triangle in $\mathbb{R}^2$ defined by the points $(1,3),(2,3),(2,7)$, determine the area of the new triangle.

We can compute the area of the triangle, which is just a right triangle with base 1 and height 4, which therefore has area 2. The matrix A will scale volumes by the absolute value of its determinant which is 9, therefore the area is 18.