Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name:___

1. (3pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps (4, 1) to (11, 3) and maps (-1, -1) to (-2, 4). Determine T((-1, 1))

We need to write (-1, 1) as a linear combination of (4, 1) and (-1, -1), which is equivalent to row reducing:

$$\begin{pmatrix} 4 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 0 & -2/3 \\ 0 & 1 & -5/3 \end{pmatrix}$$

Therefore (-1, 1) = -2/3(4, 1) - 5/3(-1, -1), so we have:

$$T((-1,1)) = -2/3T(4,1) - 5/3T(-1-1) = \frac{-2}{3} \binom{11}{3} - \frac{5}{3} \binom{-2}{4} = \frac{1}{3} \binom{-12}{-26} = \binom{-4}{-26/3}$$

2. (2pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates points $-\pi/4$ radians about the origin (clockwise). Determine the standard matrix¹ for T, call it M. [Hint: $T(\mathbf{e}_1) = (1/\sqrt{2}, -1/\sqrt{2})$]

We have $T(e_1) = (1/\sqrt{2}, -1/\sqrt{2})$ and by drawing a picture or thinking about it $T(e_2) = (1/\sqrt{2}, 1/\sqrt{2})$ Therefore the matrix we get is:

 $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

3. (2pts) Compute M^2 from problem (2) and describe what the linear map (that corresponds to this new matrix) does to points on \mathbb{R}^2

By matrix multiplication, we will get that:

$$M^2 = \begin{pmatrix} 0 & 1\\ -1 & 0 \end{pmatrix}$$

This matrix rotates points $\pi/2$ radians clockwise.

¹That is $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^2$