Please show **all** your work and circle your answer! Please read the questions carefully. You have 15 minutes for this quiz.

Name:____

1. (3pts) Determine if the columns of the following matrix are linearly independent, if they are linearly dependent, write a nontrivial linear combination of the three vectors that sums to zero:

$$\begin{pmatrix} 0 & 2 & 3 \\ 1 & 3 & 6 \\ -1 & 1 & 0 \end{pmatrix}$$

Solution: We would like to solve the equation $av_1 + bv_2 + cv_3$ with v_i the column vectors of the matrix and a, b, c constants. This is equivalent to row reducing the augmented matrix:

$$\begin{pmatrix} 0 & 2 & 3 & 0 \\ 1 & 3 & 6 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore there is a free variable, so there is a nontrivial linear combination of the three vectors summing to zero, so they are linearly dependent.

We could write $a = -3/2t$ and $b = -3/2t$ where t is our free variable. If we choose $t = 2$ (as one example) we get $a = -3, b = -3, c = 2$. And you can verify that:

$$-3\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} = \vec{0}$$

2. (3pts) From the collection of vectors in $\mathbb{R}^4$, select a subset of them that forms a basis[1] for their span (justify your answer).

$$\left\{ \begin{pmatrix} 1 \\ -3 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} -3 \\ 9 \\ -6 \\ 12 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \\ -3 \\ 7 \end{pmatrix} \right\}$$

Solution Put these vectors into a matrix, row reduce it, find the pivot columns, select the cooresponding vectors from the original matrix:

$$\begin{pmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{pmatrix} \to \begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The pivot columns are the first, third, and fourth. Therefore the following form a basis for the span of the original four vectors:

$$\left\{ \begin{pmatrix} 1 \\ -3 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \\ -3 \\ 7 \end{pmatrix} \right\}$$

[1] Recall: the basis of the span of $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a collection of linearly independent vectors that span the same space as the span of $\{\vec{v}_1, \ldots \vec{v}_n\}$.

The End.