Please show **all** your work and circle your answer! Please read the questions carefully. You can use the back of this quiz to write answers, but clearly indicate which problem you are solving. You have 15 minutes for this quiz.

Name:________________________________________

1. (3pts) Using variation of parameters, solve the following differential equation:

$$\begin{cases} y'' + y = \sec x \\ y(0) = 7 \\ y(\pi/3) = 0 \end{cases}$$

First we need to find the homogeneous solutions, which are just $y_1 = \sin(x)$ and $y_2 = \cos(x)$. The Wronskian is therefore just 1. So our particular solution is $y_1 v_1 + y_2 v_2$ with:

$$v_1 = -\int \sec(x)\cos(x) = -x$$

and:

$$v_2 = \int \sec(x)\sin(x)dx = -\ln|\cos(x)|$$

Therefore our general solution is:

$$y = c_1 \sin(x) + c_2 \cos(x) - x\sin(x) - \cos(x)\ln|\cos(x)|$$

The first initial condition requires $c_2 = 7$ and the other initial condition requires that $c_1 = \pi/3 - 8/\sqrt{3}$ (this constant wasn't super important, and may not be correct, I gave full credit if I saw that you made an attempt to find it.

2. (2pts) Write the general solution to the following system of differential equations:

$$\begin{cases} x' + 2x - y = 0 \\ y' - x + 2y = 0 \end{cases}$$

We need to put this into a matrix form differential equation. Let $X = (x, y)^t$ Then we have $X' = AX$ for:

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$$

Then $A$ has eigenvalues $-3, -1$ with eigenvectors $(1, -1)^t$ and $(1, 1)^t$. Therefore the general solution is:

$$X = c_1 e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$