MATH 54 – DISC 216, 203 PRACTICE MIDTERM OCTOBER 1ST, 2019

Please show **all** your work and circle your answer! Please read the questions carefully. You have 30 minutes.

Name:_____

1. (12 pts) For each statement, indicate whether it is true or false.
   (a) The set of all upper-triangular $n \times n$ matrices[1] is a subspace of vectorspace of square $n \times n$ matrices.
   (b) If A is an onto matrix, then the columns of A form a set of linearly independent vectors.
   (c) The map $T : \mathbb{P}_2 \to \mathbb{P}_4$ which sends $p(x) \mapsto p(x)^2$ is linear.[2]
   (d) If T is a linear map with a null space with dimension greater than 1 and $\vec{v}$ is in the image of T, then there exist more than 1 $\vec{w}$ such that $T(\vec{w}) = \vec{v}$
   (e) The column space of A is equivalent to the column space of the row reduced matrix A (put into row reduced echelon form)
   (f) $\{2, 1 - x, x^2 - x - 1\}$ forms a basis for $\mathbb{P}_2$

2. (10 pts) Are the following vectors linearly independent? If so, explain why. If not find a nontrivial linear combination that is equal to zero.

$$\begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 2 \\ -1 \end{pmatrix}$$

[1] zero below the diagonal
[2] recall that $\mathbb{P}_n$ is the vector space of all polynomials of degree less than or equal to n

3. (12 pts) Let $b_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, b_2 = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}, b_3 = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}, x = \begin{pmatrix} -8 \\ 2 \\ 3 \end{pmatrix}$.

   (a) Show that the set $\mathcal{B} = \{b_1, b_2, b_3\}$ is a basis for $\mathbb{R}^3$

   (b) Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis

   (c) Write the equation that relates x in $\mathbb{R}^3$ to $[x]_{\mathcal{B}}$

   (d) Find $[x]_{\mathcal{B}}$ for the x given above

The End.