Show all your work and circle your answers. You have 30 minutes

Name:____

1. Mark the following true or false:
   (a) Eigenvectors for distinct eigenvalues of real symmetric matrix are orthogonal
   (b) The determinant of any lower-triangular square matrix is the product of the diagonal entries
   (c) Any orthogonal matrix is diagonalizable over $\mathbb{R}$
   (d) A change of coordinates matrix is always invertible
   (e) If A is an orthogonal matrix, then the linear map $x \mapsto Ax$ is one-to-one and onto
   (f) If a 2×2 matrix is diagonalizable, then it has distinct eigenvalues.
   (g) If A is a 4×3 matrix with orthonormal columns, then $A^t A$ is the orthogonal projection matrix on $Col(A)$

2. Find a vector $v \in \mathbb{R}^2$ such that the sequence $A^n v$ has a non-zero limit as $n \to \infty$

$$A = \begin{pmatrix} 7 & 4 \\ -9 & -5 \end{pmatrix}$$

   Is A diagonalizable?

3. Find the linear function $y = \ell(x) = ax+b$ given the best fit to the data points $(x, y) = (-1, 1); (0, 1); (1, 0); (2, 2)$ in the sense of minimizing the sum of square errors $\sum(\ell(x) - y)^2$, explain your steps

4. (**3 Things:**) Compute the orthogonal projection matrix from $\mathbb{R}^3$ onto the span of the first two columns of A, for:

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

And compute A^4 and find a matrix B such that $B^3 = A$ given that the eigenvalues of A are $2, -1, -1$ with eigenvectors $(1, 1, 1)^t, (-1, 0, 1)^t, (-1, 1, 0)^t$.

The End.