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1. Major Topic: Spectral Graph Theory

Linear Algebra Preliminaries. (Spielman 2, 4) spectral theorem, Rayleigh quotients, Courant-Fischer, positive (semi-)definiteness, Perron-Frobenius, eigenvalue interlacing

Laplacians. (Trevisan 3-5, Williamson 5-7) standard / normalized Laplacian, eigenvalues and connectivity / cuts, matrix-tree theorem, edge expansion and conductance, Cheeger's Inequality

Sparse Cut Problems. (Trevisan 10-15, Matousek 4.2-4.3, Rothvoss) uniform and nonuniform sparsity, Leighton-Rao, Bourgain's theorem, Geomans-Linial/ARV

Expander Graphs. (Trevisan 16-21) characters, Cayley graphs of Abelian groups, Zig-Zag product, Marguli-Gabber-Galil Expanders, quasirandomness, random walks on expanders

Potential Theory. (Spielman 10-13) random walks on graphs, spring and resistor networks, effective resistance, Schur complements, random spanning trees

2. Major Topic: Probability Theory

Measure Theory. (Durrett 1,2) σ-algebras, Caratheodory's extension theorem, DCT, MCT, Fatou's lemma, monotone class lemma, Fubini's theorem, independence, Borel-Cantelli lemmas

Random Sequence and Series. (Durrett 2) modes of convergence, WLLN, SLLN, Glivenko-Cantelli, Portmanteau, Helly's selection theorem, tightness

Characteristic Functions and Central Limit Theorem. (Durrett 3) inversion formula, uniqueness and continuity, central limit theorem for iid sequence, Lindeberg-Feller CLT, Poisson convergence theorem

Martingales and Conditioning. (Durrett 4) Lebesgue-Radon-Nikodym theorem, conditional expectation, filtrations, stopping times, martingales, uniform integrability

Markov Chains. (LPW 4.3, 5, 1.6, 12.1-12.4, 13.1-13.3, 18.1-18.3, 20) asymptotic convergence, coupling, Wasserstein distance, reversibility, topics relating to spectra, cutoff, continuous-time Markov chains

3. Minor Topic: Analysis of Boolean Functions

Fourier expansion. (O'Donnell 1) orthonormal basis of parity functions, basic Fourier formulas, probability densities, convolution, almost linear functions, BLR test

Basic concepts and social choice. (O'Donnell 2.1-2.3) social choice functions, influences and derivatives, total influence

Majority and threshold functions. (O'Donnell 5.1, 5.2, 5.5) linear threshold functions, polynomial threshold functions, majority, CLT, Peres's theorem, uniform noise stability

Hypercontractivity. (O'Donnell 9.1, 9.6) Bonami's lemma, Kahn, Kalai, Linial theorem, Friedgut's Junta lemma