Homework 9, due Thursday October 29

1. Let \( \mathfrak{g} \) be a semisimple complex Lie algebra. A subalgebra of \( \mathfrak{g} \) is a Cartan subalgebra iff it consists entirely of semisimple elements and is maximal with respect to this property.

2. Let \( \mathfrak{g} \) be a semisimple complex Lie algebra. Show

\[
\mathfrak{b} = \mathfrak{h} \oplus \bigoplus_{\alpha > 0} \mathfrak{g}_\alpha
\]

is a maximal solvable subalgebra (called the Borel subalgebra). Show that all Borel subalgebras are equivalent.