Sample Midterm 2, Math 1A

1. Let $p \neq 0$. Show by implicit differentiation that the tangent line to the curve

$$x^p + y^p = 1, \; x > 0, \; y > 0$$

at the point (x_0, y_0) is given by the equation $x_0^{p-1}x + y_0^{p-1}y = 1$. Show that the x-intercept a and y-intercept b of the tangent line satisfy $a^{p/(1-p)} + b^{p/(1-p)} = 1$ if $p \neq 1$.

2. A ladder $10\,ft.$ long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of $2\,ft./s.$, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is $6\,ft.$ from the base of the wall?

3. Prove that $\ln(x) \leq x - 1$ for $x > 0$.

4. Let

$$g(x) = \begin{cases} e^{-1/x}, & x > 0 \\ 0, & x \leq 0 \end{cases} \tag{1}$$

Show that g is differentiable and $g'(0) = 0$.

5. Bismuth-210 has a half-life of 5.0 days. A sample of Bismuth has a mass of $128mg$.

   (a) Find a formula for the mass remaining after t days.

   (b) Find the mass remaining after 30 days.

   (c) When is the mass reduced to $1mg$?

6. Find the maxima and minima of $x^3 - 3x + 1$ on the interval $[0, 3]$.

7. Find the intervals on which f is increasing and decreasing, find the intervals of concavity and the inflection points, for the function $f(x) = (x^2 + 4x + 5)e^{-x}$.

8. Find

$$\lim_{x \to 0} \frac{x^2 \sin(1/x)}{\sin(x)},$$

or prove that the limit doesn't exist.