Final, Math 1A, section, fall 2008

1. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

2. Show that $\tan(x) > x$ for $0 < x < \pi/2$.

3. A box with a square base and open top must have a volume of $32,000 cm^3$. Find the dimensions of the box that minimizes the amount of material used.

4. Find $\frac{d}{dx}\int_{\sin x}^{\cos x} \frac{1}{\sqrt{1-t^2}}\,dt$ for $0 < x < \pi/2$, justifying your answer.

5. Show that the tangent line to the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
at the point (x_0, y_0) is
$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

6. Show that the tangent lines to the curves $x = y^3$ and $y^2 + 3x^2 = 5$ are perpendicular when the curves intersect. Justify your answer.

7. Evaluate the following integrals, justifying your answers:
   (a) $\int_0^1 x - \frac{\tan^{-1} x}{1+x^2} dx$
   (b) $\int_0^2 \sqrt{4 - x^2} dx$
   (c) $\int e^x \sqrt{1 + e^x} dx$

8. For the function $f(x) = e^x/x$, find with justification
   (a) the domain
   (b) intercepts
   (c) symmetry
   (d) asymptotes
   (e) intervals of increase or decrease
   (f) local maximum and minimum values
   (g) concavity and points of inflection
   (h) Then sketch the graph $y = f(x)$, marking on your graph all of the information you have found.

9. Let $f(x) = x - 2\sqrt{x}$.
   (a) Prove that f is increasing for $x > 1$.
   (b) Find an inverse function for $f(x)$ on the interval $x > 1$.
   (c) Prove rigorously the following limit, using the precise definition of an infinite limit:
$$\lim_{x\to\infty} x - 2\sqrt{x} = \infty$$

10. Suppose you make napkin rings by drilling holes through the centers of balls with different diameters and different sized holes. Suppose that the napkin rings have the same height h. Show that the volumes of the napkin rings are the same. Justify your answer.