Math 1A Practice Midterm 1.

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer in simplified form. There are questions on both sides of the paper. Time allowed: 1.5 hours.

1. Find the domain of the function \( f(x) = x/(3x - 1) \).

2. Sketch the graph of \( y = |\cos(x)| \) for \(-8 \leq x \leq 8\).

3. Find a formula for the inverse of the function \( f(x) = (4x - 1)/(2x + 3) \).

4. Sketch the graph of a function \( f \) that satisfies the conditions

\[
\lim_{x \to 3^+} f(x) = 1, \quad \lim_{x \to 3^-} f(x) = 2, \quad \lim_{x \to 2^+} f(x) = 0, \quad \lim_{x \to 2^-} f(x) = 1, f(2) = 1.
\]

5. Evaluate the limit

\[
\lim_{x \to 1} \sqrt{x^4 + 3x + 4}
\]

6. Find a positive number \( \delta \) such that \( |1/x - 0.5| < 0.1 \) whenever \( |x - 2| < \delta \).

7. Find the numbers at which \( f \) is discontinuous, where \( f \) is defined by

\[
\begin{align*}
f(x) &= x + 1 \text{ if } x \leq 1, \\
f(x) &= \sqrt{x - 3} \text{ if } x \geq 3.
\end{align*}
\]

8. What is

\[
\lim_{x \to +\infty} \frac{4x^2 - 3}{5x^2 - 7x + 100}
\]

9. A curve has equation \( y = f(x) \). Write and expression for the slope of the secant line through the points \((3, f(3))\) and \((x, f(x))\), and write an expression for the slope of the tangent line at \((3, f(3))\).

10. If \( g(x) = 2 - x^3 \), find \( g'(0) \) and use it to find an equation of the tangent line to the curve \( y = 2 - x^3 \) at the point \((0, 2)\).

11. Find the derivative of the function \( f(x) = x^3 \) using the definition of derivative.

12. Differentiate the function \( y = \sqrt{x - 3e^x} \)

13. Find the points on the curve \( y = 2x^3 + 3x^2 - 12x + 5 \) where the tangent is horizontal.

14. Differentiate \( x^7e^x \)

15. Differentiate

\[
\frac{\sqrt{x}}{e^x + 1}
\]

Solutions:

1. All reals other than 1/3. 2. 3. \( x = (3y + 1)/(4y - 2y) \) where \( y = f(x) \). 4. 5. \( \sqrt{3} \). Any \( \delta \) less than or equal to 1/3 will do. 7. 1 and 3 8. 4/5 9. \((f(x) - f(3))/(x - 3), \lim_{x \to 3}(f(x) - f(3))/(x - 3)\) (or \(\lim_{h \to 0}(f(3 + h) - f(3))/h\)). 10. \( g'(0) = 0 \), tangent line is \( y = 2 \). 11. \( f'(x) = \lim_{h \to 0}(x + h)^3 - x^3 = \lim_{h \to 0}3x^2h + 3xh^2 + h^3 = 3x^2 \). 12. \( x^{-1/2}/2 - 3e^x \) 13. \( x = 1 \) or \( -2 \). 14. 7x^6e^x + x^7e^x. 15. \((x^3 + 1)x^{-1/2}/2 - x^{1/2}/e^x)/(e^x + 1)^2\)