Math 1A Practice Final

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer in simplified form. There are questions on both sides of the paper.

The questions on this practice final are all exercises in Stewart; the exercise number is given at the end of the question so you can check your answer.

1. Draw the graph of \( y = \sqrt{x} + 3 \). (1.3.17)
2. Prove that \( \lim_{x \to 0} x^2 = 0 \) using the \( \epsilon, \delta \) definition of limit. (2.4.25)
3. Prove that \( e^x = 2 - x \) has at least one real root. (2.5.51)
4. Differentiate \( e^x / x^2 \). (3.2.5)
5. Find the derivative of \( \tan(\cos(x)) \). (3.5.29)
6. Find \( dy/dx \) if \( x^2 y + xy^2 = 3 \). (3.6.9)
7. Find the absolute maximum and absolute minimum values of \( x/(x^2 + 1) \) on \([0,2]\). (4.1.53)
8. Prove that \( 2 \sin^{-1}(x) = \cos^{-1}(1 - 2x^2) \) for \( 0 \leq x \leq 1 \). (4.2.32)
9. Find \( \lim_{x \to 1^+} \ln(x) \tan(\pi x/2) \). (4.4.43)
10. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side \( L \) if one side of the rectangle lies on the base of the triangle. (4.7.21)
11. Find the point on the line \( y = 4x + 7 \) that is closest to the origin. (4.7.15)
12. Use Newton’s method to find \( 30^{1/3} \) to two decimal places. (4.9.11)
13. Find the most general anti-derivative of \( 5x^{1/4} - 7x^{3/4} \). (4.10.5)
14. Find \( f \) given that \( f''(x) = 2 - 12x, f(0) = 9, f(2) = 15 \). (4.10.37)
15. Estimate the area under the graph of \( f(x) = 1 + x^2 \) from \( x = -1 \) to \( x = 2 \) using three rectangles and right endpoints. (5.1.5a)
16. Find an expression for the area under the graph of \( f(x) = x \cos(x), 0 \leq x \leq \pi/2 \) as a limit. (5.1.19)
17. Evaluate the integral \( \int_0^\pi (1 + \sqrt{9 - x^2}) \) by interpreting it as an area. (5.2.37)
18. Prove that \( \int_0^{\pi/4} \sin^3(x)dx \leq \int_0^{\pi/4} \sin^2(x)dx \). (5.2.51)
19. Find the derivative of \( g(x) = \int_0^x \sqrt{1+2t} dt \). (5.3.7)
20. Find the derivative of \( y = \int_0^{\pi/2} \sqrt{1+\sin(t)} dt \). (5.3.51)
21. Evaluate the integral \( \int_0^1 (6x^2 - 4x + 5)dx \). (5.4.17)
22. Evaluate the integral \( \int_0^{\pi/4} ((1 + \cos^2(\theta)) / \cos^2(\theta)) d\theta \). (5.4.33)
23. Evaluate the indefinite integral \( \int \frac{1}{\sqrt{1+x^2+2x}} dx \). (5.5.11)
24. Evaluate the indefinite integral \( \int \cot(x) dx \). (5.5.35)
25. Evaluate the definite integral \( \int_0^1 (x - 1)^2 dx \). (5.5.49)
26. Show that \( 1/2 + 1/3 + \cdots + 1/n < \ln(n) \). (5.6.3)
27. Find the area enclosed by the curves \( x = 2y^2, x + y = 1 \). (6.1.17)
28. Find the volume of the region obtained by rotating \( y = x^2, 0 \leq x \leq 2, y = 4, x = 0 \) about the \( y \)-axis. (6.2.5)
29. Use the method of cylindrical shells to find the volume of a sphere of radius \( r \). (6.3.43)
30. Find the average value of \( (x - 3)^2 \) on \([2, 5]\). (6.5.9a)