Math 147 Homework # 4, due 5/5/00

1. Section 2.1 problem 3.

2. Section 2.2 problems 3, 4, 7.

3. True or false, and prove: if $M_1, M_2 \subset \mathbb{R}^2$ are bounded measurable sets, and $0 < \lambda_1, \lambda_2 < 1$, then there exists a line cutting $M_i$ into two halves of volumes $\lambda_i \text{vol}(M_i)$ and $(1 - \lambda_i) \text{vol}(M_i)$. (The ham sandwich theorem says this is true for $\lambda_1 = \lambda_2 = 1/2$.)

4. Let $S_1, S_2$ be two surfaces with boundary embedded in $\mathbb{R}^3$. Suppose that each of $S_1$ and $\partial S_1$ intersects each of $S_2$ and $\partial S_2$ transversely. Show that $S_1 \cap S_2$ is a one-manifold with boundary, and

$$\partial(S_1 \cap S_2) = ((\partial S_1) \cap S_2) \cup (S_1 \cap (\partial S_2)).$$

5. Let $T = S^1 \times S^1$, and let $[X, Y]$ denote the set of homotopy classes of continuous maps from $X$ to $Y$. Recall from class that degree gives a bijection $[S^1, S^1] \simeq \mathbb{Z}$.

   (a) Establish a bijection

   $$[S^1, T^2] \simeq \mathbb{Z}^2.$$ 

   (b) (extra credit) A homotopy class $(p, q) \in \mathbb{Z}^2$ (other than $(0, 0)$) can be represented by an embedding $S^1 \to T^2$ if and only if $p$ and $q$ are relatively prime.

6. (extra credit) Let $M_1, M_2, M_3 \subset \mathbb{R}^2$ be bounded measurable sets. Show that there is a line or a circle which cuts each $M_i$ into two halves of equal volume.