Math 113 Homework # 9, due 3/16/01 at 9:00 AM

Note: this assignment is due on Friday morning, so that we can go over the problems in class as needed before the final exam. The final will be posted on Friday evening. I will hold extra office hours on Friday from 12:30-2:00 in case there are further questions.

1. Section 5.1 problem 8.
2. Let $z = (z_1, \ldots , z_n), w = (w_1, \ldots , w_n) \in \mathbb{C}^n$. Define $\langle z, w\rangle = \sum_{i=1}^n \overline{z_i} w_i$. Show that if A is an $n \times n$ complex matrix, then $\langle w, Az\rangle = \langle \overline{A}^t w, z\rangle$.
3. (a) Suppose $A \in O(n)$. Show that if λ is an eigenvalue (possibly complex) of A then $|\lambda| = 1$.
   (b) Prove: if $A \in O(3)$ and $\det(A) = 1$, then 1 is an eigenvalue of A.
   (c) True or false, and write a sentence each explaining why:
      i. If $A \in O(3)$ and $\det(A) = 1$, then A is a rotation around a line in $\mathbb{R}^3$.
      ii. If $A \in O(3)$ and $\det(A) = -1$, then A is the reflection across a plane in $\mathbb{R}^3$.
4. Section 5.2 problem 14.
5. Let A and B be symmetric $n \times n$ matrices.
   (a) Show that if A and B can be diagonalized using *the same eigenvectors*, then $AB = BA$. Hint: pick a nice basis for $\mathbb{R}^n$ and show that $ABv = BAv$ for each basis vector v.
   (b) Prove the converse. Hint: let E be an eigenspace of A, and show that B sends E to itself, so the restriction of B to E can be diagonalized.

   This result is important in quantum mechanics.
6. Section 5.3 problem 7.