Math 113 Homework # 4, due 2/9/01 at 5:00 PM

1. Let W be a subspace of an inner product space V. Show that $W^\perp$ is a subspace of V.

2. Let W be a subspace of $\mathbb{R}^n$.

   (a) Let A be a matrix whose rows are a basis for W. Show that $W^\perp = N(A)$.

   (b) Show that $\dim(W^\perp) = n - \dim(W)$.

   (c) Show that $(W^\perp)^\perp = W$. Hint: first give a simple argument that $W \subset (W^\perp)^\perp$, then use dimensional considerations.

   (d) Let $n = 4$ and $W = \text{span}((1,1,2,3),(2,3,5,8))$. Find a basis for $W^\perp$.

3. Section 2.1, problems 11,12.

4. Let W be a finite dimensional subspace of an inner product space V. Recall that for $x \in V$, the orthogonal projection $P_W(x)$ of x onto W is characterized by $P_W(x) \in W$ and $x - P_W(x) \in W^\perp$.

   (a) Show that $P_W : V \to V$ is a linear transformation.

   (b) Let $e_1, \ldots, e_m$ be an orthonormal basis for W. Show that

$$P_W(x) = \sum_{i=1}^{m} \langle x, e_i \rangle e_i.$$

5. Consider the data points $(x_1, y_1) = (-1, 1), (x_2, y_2) = (0, 2), (x_3, y_3) = (1, 3), (x_4, y_4) = (2, 5)$. Find the quadratic function $f(x) = ax^2 + bx + c$ that minimizes the "error"

$$E(a, b, c) = \sum_{i=1}^{4} (y_i - f(x_i))^2.$$

6. Section 6.2, problem 19.

7. (extra credit) When does equality hold in the triangle inequality? Prove your answer.