Math 113 Homework # 3, due 2/2/01 at 5:00 PM

Some of these problems are a little tricky. Try to think them through step by step, and don't worry if you can't get them all.

1. Section 1.3, problem 8.

2. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$.

   (a) Find a 4×2 matrix B such that $C(B) = N(A)$.

   (b) Find a 2×4 matrix B such that $N(B) = C(A)$.

3. Let V denote the space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

   (a) Show that $1, x, \ldots, x^n$ are linearly independent in V. (Hint: if a polynomial vanishes at a, then it is divisible by $x - a$, so if a polynomial of degree n vanishes at $n + 1$ points, it is zero.)

   (b) Show that $\dim(V) = \infty$.

4. Let V and W be subspaces of $\mathbb{R}^n$ with $V \cap W = \{0\}$. Show that $\dim(V) + \dim(W) \leq n$. Hint: let $v_1, \ldots, v_k$ be a basis for V, let $w_1, \ldots, w_l$ be a basis for W, and show that $v_1, \ldots, v_k, w_1, \ldots, w_l$ are linearly independent.

5. Let V be a vector space over $\mathbb{R}$ and $S \subset V$ a subset (not necessarily a subspace). Show that the following two conditions on S are equivalent:

   (a) S is nonempty; and if $x, y \in S$ and $\lambda \in \mathbb{R}$, then[1] $\lambda x + (1-\lambda) y \in S$.

   (b) There is a vector $v \in V$ and a subspace W of V, such that[2] $x \in S \Leftrightarrow x - v \in W$.

   (Such an S is called an *affine subspace* of V.)

6. Let $v_1, \ldots, v_n$ be nonzero vectors in an inner product space. Suppose that $v_i \perp v_j$ for $i \neq j$. Show that $v_1, \ldots, v_n$ are linearly independent. Hint: compute the inner product of $c_1 v_1 + \cdots + c_n v_n$ with itself.

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[1] This means that the line through any two points in S is in S.

[2] This means that S is the subspace W, translated by the vector v.