Math H185 Review

The following is a list of basic notions that we covered in the course. While I may have forgotten one or two things, if you understand all of the topics listed below and can use them in practice, then you should be well prepared for the final exam. (The last couple of lectures introduced more advanced topics which are not listed below, but which reviewed some of the topics listed below.)

- Complex numbers: basic operations and their geometric meaning, polar representation, the extended complex plane / Riemann sphere, exponential funcional, trigonometric functions, and logarithms.

- Definition of holomorphic functions, Cauchy-Riemann equations, conformal maps. When functions do not have continuous inverses, and how to fix this by modifying the domain.

- Definition of complex analytic functions. How to find the radius of convergence of a power series either directly or from the singularities of the corresponding holomorphic function. Manipulation of power series: arithmetic operations, differentiation, composition. Unique continuation of analytic functions.

- Linear fractional transformations: what they do and how to find ones with desired properties. Reflection across a line or circle.

- Differential forms on the complex plane: definition of line integrals, the d operator in its real and complex forms. Green's/Stokes's theorem. The integral of a closed form over a loop is invariant under homotopy of a loop. The integral of a closed form around a loop is determined by the winding numbers of the loop around the holes in the domain. An exact form is closed; a closed form is exact if and only if its periods are zero.

- Winding number: how to read it off from a picture or compute it by an integral. Homotopy invariance of winding number.

- Cauchy's theorem in a simply connected domain. Cauchy's integral formula, including for higher derivatives. Cauchy's theorem for non-simply-connected domains.

- Liouville's theorem, fundamental theorem of algebra.
- Isolated singularities: how to distinguish between removable singularities, poles, and essential singularities. How to compute the order of a pole or zero. How to find the singular part of a function at a pole, including the residue.
- Evaluation of definite integrals using complex analysis.
- The argument principle for counting zeroes minus poles. Integral formula for the sum of the zeroes minus the sum of the poles. Rouché's theorem.
- Open mapping theorem, maximum principle, Schwarz lemma.
- Laurent series: existence and uniqueness, where they converge, how to calculate them.
- Harmonic functions, harmonic conjugates. u has a harmonic conjugate if and only if $*du$ is exact. Mean value property. Statement of the Schwarz reflection principle for harmonic and holomorphic functions.