Name: ____________________ SID: __________ GSI: __________

7. Let $\mathbf{r}$ be a vector-valued function of t for $0 \leq t \leq 1$. Suppose that $\mathbf{r}(0) = \langle 1, 1, 1\rangle$, $\mathbf{r}(1)$ is in the x, y plane, and

$$\mathbf{r}'(t) \times \langle 1, 2, 3\rangle = 0$$

for all t. What is $\mathbf{r}(1)$?

Since $\vec{r}\,'(t) \times \langle 1,2,3\rangle = 0$, it follows that $\vec{r}\,'(t)$ is parallel to $\langle 1,2,3\rangle$. This means that $\vec{r}\,'(t) = f(t)\langle 1,2,3\rangle$ where f is a scalar function of t. By the fundamental theorem of calculus,

$$\vec{r}(1) - \vec{r}(0) = c\langle 1,2,3\rangle \quad \text{where} \quad c = \int_0^1 f(t)\,dt.$$

Writing $\vec{r}(1) = \langle x, y, 0\rangle$, this equation becomes

$$\langle x-1, y-1, -1\rangle = \langle c, 2c, 3c\rangle.$$

Equating the z components gives $c = -1/3$.
Then equating the x and y components gives
$x = 1 + c = 2/3$ and $y = 1 + 2c = 1/3$.
Thus the answer is $\boxed{\vec{r}(1) = \langle 2/3, 1/3, 0\rangle}$