

**Math 53 Midterm #1, 10/2/03, 8:10 AM – 9:30 AM**

No calculators or notes are permitted. Each of the 6 questions is worth 10 points. Please write your solution to each of the 6 questions on a separate sheet of paper with your name on it. Please put a box around the final answer and, to maximize credit, show your work. Good luck!

1. (a) Sketch the curve given in polar coordinates by the formula

$$r = e^{2\theta}, \quad 0 \leq \theta \leq 2\pi.$$

- (b) Compute the length of the above curve.

2. Show that the limit

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x + 2y - 3}{x + y - 2}$$

does not exist.

3. Suppose the function  $f(x, y)$  satisfies

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{y}.$$

Suppose as usual that  $x = r \cos \theta$  and  $y = r \sin \theta$ . Calculate  $\partial f / \partial r$  and  $\partial f / \partial \theta$  in terms of  $r$  and  $\theta$ .

4. (a) Sketch the surface  $x^2 + (y - 1)^2 = z^2$ .  
(b) Find the tangent plane to the above surface at the point  $(4, 4, 5)$ .  
(Write your answer in the form  $ax + by + cz = d$ .)

5. (a) Find normal vectors to the planes

$$2x + 2y + z = 5, \quad 2x - y - 2z = -1.$$

- (b) The above two planes intersect along a line  $L$ . At what angle do the two planes intersect? *Hint:* this is the angle between the above two normal vectors.

- (c) Find a tangent vector to the above line  $L$ . *Hint:* this must be perpendicular to the above two normal vectors.

6. Suppose  $z$  is implicitly defined as a function of  $x$  and  $y$  by the formula

$$ze^{x+2y+3z} = 4.$$

Calculate  $\partial z / \partial x$  and  $\partial z / \partial y$  in terms of  $x, y$ , and/or  $z$ .