Suggested topics for student seminar on Floer theory and pseudoholomorphic curves, Spring 2003

There is TONS to learn about in this area, and the following are a few starting points. You are welcome to go beyond this list. I did not have time to make a complete bibliography (which would have been very long), but I am happy to meet with you individually to help pick out specific papers.

- Background
  - The Nielsen-Thurston classification of surface diffeomorphisms: [3], other sources?
- Foundations of Floer theory and pseudoholomorphic curves.
  - Coherent orientations: how to count things with signs. This isn't too bad; see [6, 1].
  - Intersection positivity and adjunction formula with singularities in four dimensions; how do you prove this stuff? See [9].
  - Fredholm theory, compactness, and gluing. This is a lot of work, but it is central to proving things rigorously. There are lots of papers and a couple of books which do various parts of this in different settings. Salamon's Park City article [11] gives an introduction to these topics, and I got a hold of a good set of lecture notes from a course he taught which goes into more detail. I also have a copy of the unpublished thesis of M. Schwarz [12].
  - Virtual cycle techniques. This is hard stuff, but important for counting pseuodoholomorphic curves in more general settings. See e.g. recent papers by Fukaya-Ono, Ruan, Li-Tian, Liu-Tian, Fukaya-Oh-Ohta-Ono, McDuff, Cieliebak-Mundet-Salamon. It would be very ambitious to understand this in full detail and generality, but we could at least try to get a rough idea of how this works, and I can suggest some specific questions to try applying it to.
- Computations of symplectic Floer theory (of symplectomorphisms and Lagrangian intersections), with applications.

- A basic example which is only partially solved is to compute the Floer homology of surface symplectomorphisms, see [13, 14, 7, 4].
- Seidel's papers (available on the preprint archive) do a lot of very interesting work in this area. I also have a copy of his thesis.
- See me if you want ideas of stuff to try to compute on your own.

- Symplectic field theory.
    - The paper [5] explains the big picture; and one can get a lot out of individual sections of this paper.
    - Eliashberg's students, including F. Bourgeois, S-S. Kim, I. Ustilovsky, and M-L. Yau, have developed this theory further in their theses and proved some nice applications. I have copies of the theses of Bourgeois [2] and Ustilovsky [15] and we could try to obtain the others.
    - An interesting special case of this is the combinatorially computable "contact homology" of Legendrian knots in $\mathbb{R}^3$, which has been developed by Chekanov, Ng, and others. For the higher-dimensional version see the paper by Ekholm-Etnyre-Sullivan.

- Ozsváth-Szabó theory [10]. Perhaps this belongs in the informal topology seminar, but in any case this is an exciting new area with lots to learn and do.

- Khovanov's (purely combinatorial) categorification of the Jones polynomial [8]. It's not clear yet how this fits into the above picture, but it is intriguing.

- Most of the above is about Floer homology, but there are also many interesting papers to read about applications of pseudoholomorphic curves in closed symplectic manifolds to symplectic topology.

References

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[10] P. Ozsváth and Z. Szabó, series of recent papers at `https://math.princeton.edu/~szabo`.

[11] D. Salamon, *Lectures on Floer homology*, Symplectic geometry and topology, IAS/Park City Math Ser. 7, AMS, 1999.

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[14] P. Seidel, *Symplectic Floer homology and the mapping class group*, Pacific J. Math **206** (2002), 219–229.

[15] I. Ustilovsky, *Contact homology and contact structures on S^{4m+1}*, Stanford Univ. thesis, 1999.